Regarding Exercise 1.15

Firstly, the set $(0,1)$ is the open interval from 0 to 1. (This means the end-values 0 and 1 are excluded. If we wanted to include 0 and 1, we would write $[0,1]$.) So $(0,1)$ means the set of all real numbers strictly between 0 and 1. In set notation, we might write this:

$$(0,1) = \{a \in \mathbf{R} : 0 < a < 1\}$$

Some of you might have thought that $(0,1)$ meant the set consisting of two elements, the numbers 0 and 1. But if we meant this set, we would write this: $\{0,1\}$

Now, we have given the "name" I to the set $(0,1)$. The definition of the set J, then, references this name I. To interpret the line

$$J = \{(x_1, x_2) : x_1, x_2 \in I\}$$

and put it in sentence form, we might say (in our heads):

> J is the set of objects of the form (x_1, x_2), where x_1 and x_2 belong to I.

Or, we might say:

> J is the set of all ordered pairs of members of I.

Or:

> J is the set of all possible ordered pairs of numbers which are each constrained to lie between 0 and 1.

So here we are dealing with 2-tuples, or pairs, and again, repetition is allowed. So here are a few of the members of J:

$$\big(0.5, 0.99123\big), \big(0.3, 0.3\big), \left(\frac{1}{\sqrt{2}}, 0.1\right), \left(\frac{2}{\pi}, \frac{3\sqrt{7}}{8}\right)$$

Any other such pair you can imagine is also a member of J.

To further reinforce what this set J is, consider this alternate defintion/explanation which does not refer to I:

$$J = \{(a,b) : 0 < a < 1 \text{ and } 0 < b < 1\}$$