MATH 820: TOPICS IN NONLINEAR PDES

Optimal Transport, Gradient Flows, and Applications

Spring 2008.

Instructor: Dejan Slepčev
Time and place: MWF 11:30 in PPB 300 (CNA Seminar room)

Description: The problem of optimally transporting "material" from its location to a prescribed location, given the cost, c(x, y), of transporting a unit mass from location x to y, is a classical one. Monge studied a particular case in the 18th century. Since then optimal transport has found many applications in economics, probability, physical sciences, and elsewhere.

The course will present a mathematical description of problems of optimal transport. Existence, uniqueness, and properties of solutions will be studied. We will explore their connections to PDE's, fluid mechanics, and probability. In particular, the cost incurred by optimal transport induces a metric on the space of probability measures. Of particular interest is the Wasserstein metric that corresponds to $c(x, y) = |x - y|^2$.

For many evolution equations, arising in applications, there exists an associated dissipated quantity, that we call the energy. Often, when measured appropriately, the energy is dissipated in the optimal way. That is, the evolution is steepest descent — a gradient flow.

It turns out that in a number of important systems the dissipation mechanism has a strong connection to optimal transport. We will investigate a number of such systems — gradient flows in the Wasserstein metric. We will explore this natural viewpoint and consider applications to long-time behavior of nonlinear diffusions, geometric inequalities, and scaling laws in energy-driven systems.