Errata for  
*Stochastic Calculus for Finance II*  
*Continuous-Time Models*  
September 2006

**Page 6, lines 1, 3 and 7 from bottom.** Replace $A_{n,m}$ by $S_{n,m}$.

**Page 21, line 12.** After “Borel measurable.” insert the sentence

Throughout this text, without further mention every function we consider is assumed to be Borel measurable.

**Page 21, last line.** Move the equation

$$\int_{\mathbb{R}} f(x) \, dL(x) = \int_{\mathbb{R}} f^+(x) \, dL(x) - \int_{\mathbb{R}} f^-(x) \, dL(x),$$

to the top of page 22.

**Page 22, first line.** This page should begin with the equation

$$\int_{\mathbb{R}} f(x) \, dL(x) = \int_{\mathbb{R}} f^+(x) \, dL(x) - \int_{\mathbb{R}} f^-(x) \, dL(x),$$

moved from the bottom of page 21.

**Page 21, last line.** Move the text “Theorem 1.3.8(i) may be restated as:” to the top of page 23.

**Page 23, first line.** This page should begin with the text “Theorem 1.3.8(i) may be restated as:” moved from the bottom of page 22.

**Page 36, line 6.** Replace $\tilde{E}Z$ by $EZ$.

**Page 47, line 4.** Replace $\tilde{P}(A \bar{\omega})$ by $\tilde{P}(A(\omega, \epsilon))$.

**Page 55, line 2.** Change “Figure 1.2.2” to “Example 1.2.2.”

**Page 70, line 9.** Replace “sub-σ algebra” by “sub-σ-algebra.”

**Page 72, line 12.** After “Chapter 2” insert “of Volume I.”

**Page 73, lines 1 and 2 from bottom.** The equation should be

$$g(x) = \mathbb{E} f \left( x, \frac{\rho \sigma_2 x + W}{\sigma_1} \right)$$

$$= \frac{1}{\sigma_3 \sqrt{2\pi}} \int_{-\infty}^{\infty} f \left( x, \frac{\rho \sigma_2 x + w}{\sigma_1} \right) \exp \left\{ -\frac{(w - \mu_3)^2}{2\sigma_3^2} \right\} \, dw.$$

**Page 78, line 14.** Change “Example 2.2.8” to “Example 2.2.10.”

**Page 80, line 5.** Remove the text “Let $X$ be a random variable.”

**Page 93, line 14.** The left-hand side of the equation should be $\log S_n(t)$.

**Page 102, line 1.** Change the sentence to, “We usually work with functions that have continuous derivatives, and their quadratic variations are zero.”
Page 105, last line. On the right-hand side of the inequality, $W(k)$ should be $W(t_k)$.

Page 113, equation (3.7.4). There are two places where the exponent $\alpha m$ should be $\alpha t$. The equation should be

$$\mathbb{E} e^{-\alpha \tau_m} = \int_0^\infty e^{-\alpha t} f_{\tau_m}(t) \, dt = \int_0^\infty \frac{|m|}{t\sqrt{2\pi t}} e^{-\alpha t - \frac{m^2}{2t}} \, dt \text{ for all } \alpha > 0. \tag{3.7.4}$$

Page 116, line 12. The equation should be

$$f_{\tau_m}(t) = \frac{|m|}{t\sqrt{2\pi t}} e^{-\frac{m^2}{2t}}.$$

Page 118, line 1. Change $m$ to $n$. The text should be “... as the number $n$ of partition points ....”

Page 119, line 16. Change $h(y)$ to $f(y)$, so the equation is $g(x) = \int_0^\infty f(y) p(\tau, x, y) \, dy$.

Pages 122 and 123, Exercise 3.9. Replace with the following exercise:

Exercise 3.9 (Laplace transform of first passage density; solution provided by Kaiping Chen and Ji Li). Let $m > 0$ be given and define

$$f(t) = \frac{m}{t\sqrt{2\pi t}} \exp\left\{ - \frac{m^2}{2t} \right\}.$$

According to (3.7.3) in Theorem 3.7.1, $f(t)$ is the density of the first passage time

$$\tau_m = \min\{t \geq 0; W(t) = m\},$$

where $W$ is a Brownian motion without drift. Let

$$g(\alpha) = \int_0^\infty e^{-\alpha t} f(t) \, dt, \quad \alpha > 0,$$

be the Laplace transform of the density $f(t)$. This problem verifies directly, without resort to the probabilistic arguments of this chapter, that

$$g(\alpha) = e^{-m\sqrt{2\pi\alpha}}, \quad \alpha > 0,$$

which is the formula derived in Theorem 3.6.2.

(i) For positive numbers $a$ and $b$, define

$$I(a, b) = \int_0^\infty \exp\left\{ -a^2 x^2 - \frac{b^2}{x^2} \right\} \, dx.$$

Make the change of variable $y = b/(ax)$ to show that

$$I(a, b) = \frac{b}{a} \int_0^\infty \frac{1}{y^2} \exp\left\{ -a^2 y^2 - \frac{b^2}{y^2} \right\} \, dy$$

$$= \frac{b}{a} \int_0^\infty \frac{1}{x^2} \exp\left\{ -a^2 x^2 - \frac{b^2}{x^2} \right\} \, dx.$$
(ii) Sum the two equations for $I(a, b)$ in part (i) and divide by 2 to obtain

$$I(a, b) = \frac{1}{2a} \int_{0}^{\infty} \left( a + \frac{b}{x^{2}} \right) \exp \left\{ -ax^{2} - \frac{b^{2}}{x^{2}} \right\} dx.$$ 

Make the change of variable $t = ax - b/x$ and show that

$$I(a, b) = \frac{\sqrt{\pi}}{2a} e^{-2ab}.$$ 

(Hint: Consider the normal density with mean zero and variance $1/2$.)

(iii) Make the change of variable $x = t^{-1/2}$ in the definition of $g(\alpha)$ and conclude from (ii) that

$$g(\alpha) = \frac{2m}{\sqrt{2\pi}} I(m/\sqrt{2}, \sqrt{\alpha}) = e^{-m\sqrt{2\alpha}}.$$ 

Page 141, line 5 from bottom. Change $f_{xx}$ to $f_{tt}$. The line should be

$$+ f_{tx}(t, W(t)) \, dt \, dW(t) + \frac{1}{2} f_{tt}(t, W(t)) \, dt \, dt.$$ 

Page 144, line 6 from bottom. Change $0 \cdot \int_{0}^{L} |\Theta(u)|^2 \, du = 0$ to $0 \cdot \int_{0}^{L} |\Theta(u)| \, du = 0$.

Page 146, line 12. Change (4.4.19) to (4.4.21).

Page 162, line 10 from bottom. Change $f(t, S(0))$ to $f(0, S(0))$.

Page 162, line 9 from bottom. Change text to “... set up a static hedge, which is a hedge that does not trade....”

Page 170, line 7. Insert $\frac{1}{2}$ before $f_{yy}$. The line should be

$$\frac{1}{2} f_{xx} \, dM_1 \, dM_1 + f_{xy} \, dM_1 \, dM_2 + \frac{1}{2} f_{yy} \, dM_2 \, dM_2.$$ 

Page 187, line 11 from bottom. Change $\int_{0}^{T} \Delta^2(t) \, dW(t)$ to $\int_{0}^{T} \Delta(t) \, dW(t)$.

Page 187, line 8 from bottom. There is a $dt$ missing in the integral. The line should be

$$\int_{0}^{T} \Delta^2(t) \, dt < \infty \text{ almost surely.}$$ 

Page 196, equation (4.10.20). The partial derivatives should be with respect to $x$, not $s$. The equation should be

$$c_t(t, S(t)) + rS(t)c_x(t, S(t)) + \frac{1}{2} \sigma^2 S(t)c_{xx}(t, S(t)) = rc(t, S(t)).$$ 

(4.10.20)

Page 200, line 1. A $dt$ is missing in the equation. It should be $dB_i(t) \, dB_k(t) = \rho_{ik}(t) \, dt$.

Page 201, line 9. A $dt$ is missing in the equation. It should be $dB_1(t) \, dB_2(t) = \rho(t) \, dt$.
Page 202, equation (4.10.32). E should be $E$.

Page 203, last two lines. The label (4.10.39) should be on the last line, not the next-to-last line.

Page 207, line 13 from bottom. The line should be “level $K$ before time $T$ are those for which $L_K(T) > 0$.”

Page 222, line 11. $\bar{E} \int_0^T \Theta^2(u)Z^2(u) du < \infty$ should be $E \int_0^T \Theta^2(u)Z^2(u) du < \infty$.

Page 224, lines 15–16. “Observeed” should be “observed.”

Page 246, line 14. The line should be “or borrowing at the interest rate $R$ as necessary, satisfies...” The interest rate $R$ should be capitalized.

Page 250, line 7 from bottom. And $\exp$ is missing. The equation should be

$$E \exp \left\{ \frac{1}{2} \int_0^T \Theta^2(u) du \right\} < \infty.$$ 

Page 253, line 6. $dS(t)$ is missing on the right-hand side. The equation should be

$$dS(t) = r(t)S(t) dt + \sigma(t)S(t) d\tilde{W}(t).$$

Page 253, line 10. E should be $\bar{E}$ on the right-hand side of the equation.

Page 253, line 11 from bottom. The right-hand side of the equation should be

$$BSM \left( T, S(0); K, \frac{1}{T} \int_0^T r(t) dt, \sqrt{\frac{1}{T} \int_0^T \sigma^2(t) dt} \right).$$

Page 254, line 8 from bottom. $d\tilde{B}(u)$ on the right-hand side of the equation should be $d\tilde{W}(u)$.

Page 265, lines 9, 11 and 14 from bottom. $\alpha(u)$ should be $a(u)$.

Page 266, line 3 $\alpha(u)$ should be $a(u)$.

Page 291, equation (6.9.47). $\beta(t,y)$ should be $\beta(T,y)$.

Page 292, line 10. There is a $du$ missing. The line should be

$$\frac{1}{2} \int_t^T \int_0^b \gamma^2(u,y)p(t,u,x,y)h^y_u(y)dydu.$$

Page 324, line 5 from bottom. The first $S(t)$ should be $dS(t)$. The line should be

$$e^{r(T-t)} \gamma(t)(dS(t) - rS(t) dt).$$

Page 325, line 11. A $dt$ is missing. The equation in the middle of the line should be $d\gamma(t) = -\frac{T}{c} e^{-r(T-t)} dt$.

Page 326, line 2. “Exlain” should be “explain.”

Page 331, lines 7, 10, and 12. Replace “lookback call” by “lookback option” in three places.
Page 343, line 9 from bottom. Replace “and an $H$ on the second toss” by “and a $T$ on the second toss.”

Page 348, last line. There is a $t$ missing on the right-hand side. The equation should be

$$S(t) = x \exp \left\{ \sigma \widetilde{W}(t) + \left( r - \frac{1}{2} \sigma^2 \right) t \right\}.$$

Page 353, equation (8.3.21). $\mathbb{I}_{\{S(t) < L^*\}}$ should be $\mathbb{I}_{\{S(t) < L_*\}}$. The $*$ should be a subscript on $L$, not a superscript.

Page 354, lines 5 and 6 from bottom. $S(t) < L^*$ should be $S(t) < L_*$ in two places. The $*$ should be a subscript on $L$, not a superscript.

Page 360, equation (8.4.15). This should be an inequality. It should be

$$e^{-rt} v(t, x) \geq \mathbb{E} \left[ e^{-r\tau} (K - S(\tau)) \right| S(t) = x].$$

Page 360, line 2 from bottom. Change “for any $\tau \in \mathcal{T}_t, T$” to “for every $\tau \in \mathcal{T}_t, T$.”

Page 361, line 9 from bottom. Remove “nonnegative.” The sentence should be “Let $h(x)$ be a convex function of $x \geq 0$ satisfying $h(0) = 0$.

Page 365, equation (8.5.17). $c_n(t, x)$ on the left-hand side of the equation should be $c_n(T, x)$.

Page 396, line 10 from bottom. $\mathbb{P}\{S(T, T) > K\}$ should be $\mathbb{P}\{S(T, T) > K\}.$

Page 400, line 9 from bottom. $\tilde{W}_1(t)$ and $\tilde{W}_2(t)$ should be $\tilde{W}^{(N)}_1(t)$ and $\tilde{W}^{(N)}_2(t)$.

Page 403, equation (10.1.1). The lower limit of summation should be $i = 1$. The equation should be

$$\sum_{i=1}^{j} C_i B(0, T_i).$$

Page 406, equation (10.2.2). The left-hand side of the equation should be $dX_2(t)$, not $dX_1(t)$.

Page 412, line 6 from bottom. $\lambda$ should be $\lambda_1$, so the expression is $C'_1 + \lambda_1 C_1 + \lambda_2 C_2 - \delta_1$.

Page 416, equation (10.2.34). A $dt$ is missing. The equation should be

$$dY(t) = -AY(t) dt + d\tilde{W}(t).$$

Page 429, line 7. A $dt$ is missing. The line should be

$$\sigma(t, T) \sigma^{\ast}(t, T) dt + \sigma(t, T) \left[ \Theta(t) dt + dW(t) \right].$$

Page 437, line 13 from bottom. Replace $T$ by $T + \delta$. The line should be “Let $0 \leq t \leq T + \delta$ and $\delta > 0$ be given.”
Page 453, equation (10.7.4). The equation should be
\[ C'_1 = -\lambda_1 C_1 - \frac{1}{2} C_1^2 - \sigma_{21} C_1 C_2 - \frac{1}{2} (\sigma_{21}^2 + \beta) C_2^2 + \delta_1. \] (10.7.4)

Page 454, line 9. There is a missing comma. The text should be “model parameters $\lambda_1 > 0$, $\lambda_2 > 0$, $\lambda_{21}$, $\delta_1$, and $\delta_2$,...”

Page 457, equation (10.7.18). $C_1$ should be $C_j$. The equation should be
\[ \tilde{W}_j^T(t) = \int_0^t C_j(T - u) du + \tilde{W}_j(t), \quad j = 1, 2. \] (10.7.18)

Page 457, line 14. The second $Y_1(T)$ should be $Y_2(T)$. The equation should be
\[ X = -C_1(T - T)Y_1(T) - C_2(T - T)Y_2(T) - A(T - T). \]

Page 470, lines 5 and 12 from bottom. Change “moment generating” to “moment-generating.”

Page 470. The last line should be
\[ = \mathbb{P}\{N(t) = 0\} + \sum_{k=1}^{\infty} \mathbb{E}\left[ \exp\left\{ u \sum_{i=1}^{k} Y_i \right\} \mid N(t) = k \right] \mathbb{P}\{N(t) = k\}. \]

Page 520, line 8 from bottom. The line should be
\[ + \int_0^t e^{-ru} t \left[ \sum_{m=1}^{M} \tilde{p}(y_m) c(u, (y_m + 1)S(u)) - c(u, S(u)) \right] du. \]

Page 521, line 15. $y + 1$ should be $y_m + 1$, so the line is
\[ -e^{-rt} t \left[ \sum_{m=1}^{M} \tilde{p}(y_m) c(t, (y_m + 1)S(t-)) - c(t, S(t-)) \right] dt. \] (11.7.36)

Page 521, line 11 from bottom. The lower limit of summation should be $m = 1$, so the equation is $N(t) = \sum_{m=1}^{M} N_m(t)$.

Page 521, line 10 from bottom. The lower limit in the sum should be $m = 1$, so the sum is $\sum_{m=1}^{M} \tilde{p}(y_m) c(t, (y_m + 1)S(t-))$. There is a left parenthesis missing before the $y$ in the integrand of the integral; the integral should be $\int_{-\infty}^{\infty} c(t, (y + 1)S(t-)) f(y) dy$. Put a period at the end of the line.

Page 522, line 3. The $\lambda$ in $\tilde{\beta}\lambda t$ at the end should be $\tilde{\lambda}$. The line should be
\[ = e^{-rt} \left[ \Gamma(t) \sigma S(t) d\tilde{W}(t) + \Gamma(t-) S(t-) dQ(t) - \tilde{\beta}\lambda t \right]. \]