Errata for
Stochastic Calculus for Finance II
Continuous-Time Models
September 2006

Page 6, lines 1, 3 and 7 from bottom. Replace $A_{n,m}$ by $S_{n,m}$.
Page 21, line 12. After "*Borel measurable.*" insert the sentence
Throughout this text, without further mention every function we consider is assumed to be Borel measurable.
Page 21, last line. Move the equation

$$\int_{\mathbb{R}} f(x)\, d\mathcal{L}(x) = \int_{\mathbb{R}} f^+(x)\, d\mathcal{L}(x) - \int_{\mathbb{R}} f^-(x)\, d\mathcal{L}(x),$$

to the top of page 22.
Page 22, first line. This page should begin with the equation

$$\int_{\mathbb{R}} f(x)\, d\mathcal{L}(x) = \int_{\mathbb{R}} f^+(x)\, d\mathcal{L}(x) - \int_{\mathbb{R}} f^-(x)\, d\mathcal{L}(x),$$

moved from the bottom of page 21.
Page 21, last line. Move the text "Theorem 1.3.8(i) may be restated as:" to the top of page 23.
Page 23, first line. This page should begin with the text "Theorem 1.3.8(i) may be restated as:" moved from the bottom of page 22.
Page 36, line 6. Replace $\widetilde{\mathbb{E}}Z$ by $\mathbb{E}Z$.
Page 47, line 4. Replace

$$\frac{\widetilde{\mathbb{P}}(A)}{\mathbb{P}(A)} \quad \text{by} \quad \frac{\widetilde{\mathbb{P}}\big(A(\overline{\omega},\epsilon)\big)}{\mathbb{P}\big(A(\overline{\omega},\epsilon)\big)}.$$

Page 55, line 2. Change "Figure 1.2.2" to "Example 1.2.2."
Page 70, line 9. Replace "sub-σ algebra" by "sub-σ-algebra."
Page 72, line 12. After "Chapter 2" insert "of Volume I."
Page 73, lines 1 and 2 from bottom. The equation should be

$$\begin{aligned} g(x) &= \mathbb{E} f\left(x, \frac{\rho\sigma_2}{\sigma_1}x + W\right) \\ &= \frac{1}{\sigma_3\sqrt{2\pi}} \int_{-\infty}^{\infty} f\left(x, \frac{\rho\sigma_2}{\sigma_1}x + w\right) \exp\left\{-\frac{(w-\mu_3)^2}{2\sigma_3^2}\right\} dw. \end{aligned}$$

Page 78, line 14. Change "Example 2.2.8" to "Example 2.2.10."
Page 80, line 5. Remove the text "Let X be a random variable."
Page 93, line 14. The left-hand side of the equation should be $\log S_n(t)$.
Page 102, line 1. Change the sentence to, "We usually work with functions that have continuous derivatives, and their quadratic variations are zero."

Page 105, last line. On the right-hand side of the inequality, $W(k)$ should be $W(t_k)$.

Page 113, equation (3.7.4). There are two places where the exponent αm should be αt. The equation should be

$$\mathbb{E}e^{-\alpha\tau_m} = \int_0^\infty e^{-\alpha t} f_{\tau_m}(t)\,dt = \int_0^\infty \frac{|m|}{t\sqrt{2\pi t}} e^{-\alpha t - \frac{m^2}{2t}}\,dt \text{ for all } \alpha > 0. \tag{3.7.4}$$

Page 116, line 12. The equation should be

$$f_{\tau_m}(t) = \frac{|m|}{t\sqrt{2\pi t}} e^{-\frac{m^2}{2t}}.$$

Page 118, line 1. Change m to n. The text should be "... as the number n of partition points ...."

Page 119, line 16. Change $h(y)$ to $f(y)$, so the equation is $g(x) = \int_0^\infty f(y)p(\tau,x,y)\,dy$.

Pages 122 and 123, Exercise 3.9. Replace with the following exercise:

Exercise 3.9 (Laplace transform of first passage density; solution provided by Kaiping Chen and Ji Li). Let $m > 0$ be given and define

$$f(t) = \frac{m}{t\sqrt{2\pi t}} \exp\left\{-\frac{m^2}{2t}\right\}.$$

According to (3.7.3) in Theorem 3.7.1, $f(t)$ is the density of the first passage time

$$\tau_m = \min\{t \geq 0; W(t) = m\},$$

where W is a Brownian motion without drift. Let

$$g(\alpha) = \int_0^\infty e^{-\alpha t} f(t)\,dt, \quad \alpha > 0,$$

be the Laplace transform of the density $f(t)$. This problem verifies directly, without resort to the probabilistic arguments of this chapter, that

$$g(\alpha) = e^{-m\sqrt{2\alpha}}, \quad \alpha > 0,$$

which is the formula derived in Theorem 3.6.2.

(i) For positive numbers a and b, define

$$I(a,b) = \int_0^\infty \exp\left\{-a^2x^2 - \frac{b^2}{x^2}\right\} dx.$$

Make the change of variable $y = b/(ax)$ to show that

$$\begin{aligned} I(a,b) &= \frac{b}{a}\int_0^\infty \frac{1}{y^2} \exp\left\{-a^2y^2 - \frac{b^2}{y^2}\right\} dy \\ &= \frac{b}{a}\int_0^\infty \frac{1}{x^2} \exp\left\{-a^2x^2 - \frac{b^2}{x^2}\right\} dx. \end{aligned}$$

(ii) Sum the two equations for $I(a,b)$ in part (i) and divide by 2 to obtain

$$I(a,b) = \frac{1}{2a}\int_0^\infty \left(a + \frac{b}{x^2}\right)\exp\left\{-ax^2 - \frac{b^2}{x^2}\right\}dx.$$

Make the change of variable $t = ax - b/x$ and show that

$$I(a,b) = \frac{\sqrt{\pi}}{2a}e^{-2ab}.$$

(Hint: Consider the normal density with mean zero and variance 1/2.)

(iii) Make the change of variable $x = t^{-1/2}$ in the definition of $g(\alpha)$ and conclude from (ii) that

$$g(\alpha) = \frac{2m}{\sqrt{2\pi}} I\big(m/\sqrt{2}, \sqrt{\alpha}\big) = e^{-m\sqrt{2\alpha}}.$$

Page 141, line 5 from bottom. Change f_{xx} to f_{tt}. The line should be

$$+f_{tx}\big(t, W(t)\big)\,dt\,dW(t) + \frac{1}{2}f_{tt}\big(t, W(t)\big)\,dt\,dt.$$

Page 144, line 6 from bottom. Change $0 \cdot \int_0^t |\Theta(u)|^2\,du = 0$ to $0 \cdot \int_0^t |\Theta(u)|\,du = 0$.

Page 146, line 12. Change (4.4.19) to (4.4.21).

Page 162, line 10 from bottom. Change $f(t, S(0))$ to $f(0, S(0))$.

Page 162, line 9 from bottom. Change text to "... set up a *static hedge*, which is a hedge that does not trade...."

Page 170, line 7. Insert $\frac{1}{2}$ before f_{yy}. The line should be

$$\frac{1}{2}f_{xx}\,dM_1\,dM_1 + f_{xy}\,dM_1\,dM_2 + \frac{1}{2}f_{yy}\,dM_2\,dM_2.$$

Page 187, line 11 from bottom. Change $\int_0^T \Delta^2(t)\,dW(t)$ to $\int_0^T \Delta(t)\,dW(t)$.

Page 187, line 8 from bottom. There is a dt missing in the integral. The line should be

$$\int_0^T \Delta^2(t)\,dt < \infty \text{ almost surely.}$$

Page 196, equation (4.10.20). The partial derivatives should be with respect to x, not s. The equation should be

$$c_t\big(t, S(t)\big) + rS(t)c_x\big(t, S(t)\big) + \frac{1}{2}\sigma^2 S^2(t)c_{xx}\big(t, S(t)\big) = rc\big(t, S(t)\big). \tag{4.10.20}$$

Page 200, line 1. A dt is missing in the equation. It should be $dB_i(t)\,dB_k(t) = \rho_{ik}(t)\,dt$.

Page 201, line 9. A dt is missing in the equation. It should be $dB_1(t)\,dB_2(t) = \rho(t)\,dt$.

Page 202, equation (4.10.32). E should be $\mathbb{E}$.
Page 203, last two lines. The label (4.10.39) should be on the last line, not the next-to-last line.
Page 207, line 13 from bottom. The line should be "level K before time T are those for which $L_K(T) > 0$.)".
page 222, line 11. $\widetilde{\mathbb{E}} \int_0^T \Theta^2(u) Z^2(u)\, du < \infty$ should be $\mathbb{E} \int_0^T \Theta^2(u) Z^2(u)\, du < \infty$.
Page 224, lines 15–16. "Observeed" should be "observed."
Page 246, line 14. The line should be "or borrowing at the interest rate R as necessary, satisfies...." The interest rate R should be capitalized.
Page 250, line 7 from bottom. And *exp* is missing. The equation should be

$$\mathbb{E} \exp \left\{ \frac{1}{2} \int_0^T \Theta^2(u)\, du \right\} < \infty.$$

Page 253, line 6. And $S(t)$ is missing on the right-hand side. The equation should be

$$dS(t) = r(t)S(t)\, dt + \sigma(t)S(t)\, d\widetilde{W}(t).$$

Page 253, line 10. $\mathbb{E}$ should be $\widetilde{\mathbb{E}}$ on the right-hand side of the equation.
Page 253, line 11 from bottom. The right-hand side of the equation should be

$$\text{BSM} \left(T, S(0); K, \frac{1}{T} \int_0^T r(t)\, dt, \sqrt{\frac{1}{T} \int_0^T \sigma^2(t)\, dt} \right).$$

Page 254, line 8 from bottom. $d\widetilde{B}(u)$ on the right-hand side of the equation should be $d\widetilde{W}(u)$.
Page 265, lines 9, 11 and 14 from bottom. $\alpha(u)$ should be $a(u)$.
Page 266, line 3 $\alpha(u)$ should be $a(u)$.
Page 291, equation (6.9.47). $\beta(t, y)$ should be $\beta(T, y)$.
Page 292, line 10. There is a du missing. The line should be

$$\frac{1}{2} \int_t^T \int_0^b \gamma^2(u, y) p(t, u, x, y) h_b''(y) dy du.$$

Page 324, line 5 from bottom. The first $S(t)$ should be $dS(t)$. The line should be

$$= e^{r(T-t)} \gamma(t) \big(dS(t) - rS(t)\, dt \big).$$

Page 325, line 11. A dt is missing. The equation in the middle of the line should be $d\gamma(t) = -\frac{1}{c} e^{-r(T-t)}\, dt$.
Page 326, line 2. "Exlain" should be "explain."
Page 331, lines 7, 10, and 12. Replace "lookback call" by "lookback option" in three places.

Page 343, line 9 from bottom. Replace "and an H on the second toss" by "and a T on the second toss."

Page 348, last line. There is a t missing on the right-hand side. The equation should be

$$S(t) = x \exp\left\{\sigma \widetilde{W}(t) + \left(r - \frac{1}{2}\sigma^2\right) t\right\}.$$

Page 353, equation (8.3.21). $\mathbb{I}_{\{S(t)<L^*\}}$ should be $\mathbb{I}_{\{S(t)<L_*\}}$. The $*$ should be a subscript on L, not a superscript.

Page 354, lines 5 and 6 from bottom. $S(t) < L^*$ should be $S(t) < L_*$ in two places. The $*$ should be a subscript on L, not a superscript.

Page 360, equation (8.4.15). This should be an inequality. It should be

$$e^{-rt}v(t,x) \geq \widetilde{\mathbb{E}}\big[e^{-r\tau}\big(K - S(\tau)\big)\big|S(t) = x\big]. \tag{8.4.15}$$

Page 360, line 2 from bottom. Change "for any $\tau \in \mathcal{T}_{t,T}$" to "for every $\tau \in \mathcal{T}_{t,T}$.

Page 361, line 9 from bottom. Remove "nonnegative." The sentence should be "Let $h(x)$ be a convex function of $x \geq 0$ satisfying $h(0) = 0$.

Page 365, equation (8.5.17). $c_n(t,x)$ on the left-hand side of the equation should be $c_n(T,x)$.

Page 396, line 10 from bottom.
$\mathbb{P}\{\text{For}_S(T,T) > K\}$ should be $\widetilde{\mathbb{P}}^T\{\text{For}_S(T,T) > K\}$.

Page 400, line 9 from bottom. $\widetilde{W}_1(t)$ and $\widetilde{W}_2(t)$ should be $\widetilde{W}_1^{(N)}(t)$ and $\widetilde{W}_2^{(N)}(t)$.

Page 403, equation (10.1.1). The lower limit of summation should be $i = 1$. The equation should be

$$\sum_{i=1}^{j} C_i B(0, T_i). \tag{10.1.1}$$

Page 406, equation (10.2.2). The left-hand side of the equation should be $dX_2(t)$, not $dX_1(t)$.

Page 412, line 6 from bottom. λ should be λ_1, so the expression is $C_1' + \lambda_1 C_1 + \lambda_{21} C_2 - \delta_1$.

Page 416, equation (10.2.34). A dt is missing. The equation should be

$$dY(t) = -\Lambda Y(t)\,dt + d\widetilde{W}(t). \tag{10.2.23}$$

Page 429, line 7. A dt is missing. The line should be

$$\sigma(t,T)\sigma^*(t,T)\,dt + \sigma(t,T)\Big[\Theta(t)\,dt + dW(t)\Big].$$

Page 437, line 13 from bottom. Replace T by $T + \delta$. The line should be "Let $0 \leq t \leq T + \delta$ and $\delta > 0$ be given."

Page 453, equation (10.7.4). The equation should be

$$C_1' = -\lambda_1 C_1 - \frac{1}{2}C_1^2 - \sigma_{21}C_1C_2 - \frac{1}{2}(\sigma_{21}^2 + \beta)C_2^2 + \delta_1. \tag{10.7.4}$$

Page 454, line 9. There is a missing comma. The text should be "model parameters $\lambda_1 > 0$, $\lambda_2 > 0$, λ_{21}, δ_1, and δ_2...."

Page 457, equation (10.7.18). C_1 should be C_j. The equation should be

$$\widetilde{W}_j^T(t) = \int_0^t C_j(T-u)\,du + \widetilde{W}_j(t), \quad j = 1, 2. \tag{10.7.18}$$

Page 457, line 14. The second $Y_1(T)$ should be $Y_2(T)$. The equation should be

$$X = -C_1(\overline{T} - T)Y_1(T) - C_2(\overline{T} - T)Y_2(T) - A(\overline{T} - T).$$

Page 470, lines 5 and 12 from bottom. Change "moment generating" to "moment-generating."

Page 470. The last line should be

$$= \mathbb{P}\{N(t) = 0\} + \sum_{k=1}^{\infty} \mathbb{E}\left[\exp\left\{u\sum_{i=1}^{k} Y_i\right\}\middle| N(t) = k\right]\mathbb{P}\{N(t) = k\}.$$

Page 520, line 8 from bottom. The line should be

$$+\int_0^t e^{-ru}\tilde{\lambda}\Big[\sum_{m=1}^{M}\tilde{p}(y_m)c\big(u, (y_m+1)S(u)\big) - c\big(u, S(u)\big)\Big]du.$$

Page 521, line 15. $y+1$ should be y_m+1, so the line is

$$-e^{-rt}\tilde{\lambda}\left[\sum_{m=1}^{M}\tilde{p}(y_m)c\big(t, (y_m+1)S(t-)\big) - c\big(t, S(t-)\big)\right]dt. \tag{11.7.36}$$

Page 521, line 11 from bottom. The lower limit of summation should be $m = 1$, so the equation is $N(t) = \sum_{m=1}^{M} N_m(t)$.

Page 521, line 10 from bottom. The lower limit in the sum should be $m = 1$, so the sum is $\sum_{m=1}^{M}\tilde{p}(y_m)c(t, (y_m+1)S(t-))$. There is a left parenthesis missing before the y in the integrand of the integral; the integral should be $\int_{-1}^{\infty} c(t, (y+1)S(t-))\widetilde{f}(y)\,dy$. Put a period at the end of the line.

Page 522, line 3. The λ in $\tilde{\beta}\lambda t$ at the end should be $\tilde{\lambda}$. The line should be

$$= e^{-rt}\big[\varGamma(t)\sigma S(t)\,d\widetilde{W}(t) + \varGamma(t-)S(t-)d(Q(t) - \tilde{\beta}\tilde{\lambda}t)\big].$$