# SYLLABUS FOR (FORMALLY) MATH 21 − 882 : ADVANCED TOPICS IN FINANCIAL MATHEMATICS - LARGE DEVIATIONS PRINCIPLES

Jean-Dominique Deuschel, Daniel W. Stroock in the preface to *Large Deviations*:

There is no more a "theory" of large deviations than there is a "theory" of partial differential equations; and what passes for the "theory" is, in reality, little more than a grab-bag of techniques which have been successfully applied to special situations.... In spite of the preceeding admission ... a useful purpose can be served by a book which surveys a few outstanding successes and attempts to codify some of the principles on which those success are based.

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**Lectures:** Thursday 4:30 PM to 6:30 PM in Wean Hall room 6423.

**Recitation Sessions:** Wednesday 10:00 AM to 12:00 PM in my office.

**Course Website :** www.math.cmu.edu/users/scottrob/teaching/LDP

**Textbook(s):** The primary textbook is:

*Large Deviations Techniques and Applications, 2nd Edition*
Amir Dembo, Ofer Zeitouni
Springer

Three other textbooks which will be used to varying degrees:

*Large Deviations*
Jean-Dominique Deuschel, Daniel W. Stroock
AMS Chelsea

*A Weak Convergence Approach to the Theory of Large Deviations*
Paul Dupuis, Richard S. Ellis
Wiley

*Large Deviations*
Frank den Hollander
Fields Institute Monographs #14
AMS Providence

**Homework:** Exercises will be given during lecture and should be completed (or at least sincerely attempted) by the following week.

**Course Objectives:** This course provides an introduction to the theory of Large Deviations emphasizing applications to Financial Mathematics where appropriate. Large Deviations Principles (LDP) will be derived for empirical averages of random variables, for random walks and diffusions

and for occupancy times of Markov processes. Basic definitions, as well as general techniques for proving the existence of LDP, will be covered.

## Course Schedule (Tentative)

January 21 : Introduction, Basic Definitions, Cramér's Theorem in $\mathbb{R}^d$.

January 28 : Cramér's Theorem in $\mathbb{R}^d$ (continued) and applications.

February 4 : Gärtner-Ellis Theorem in $\mathbb{R}^d$ with an application to Importance Sampling for Portfolio Credit Risk.

February 11 : General Principles - Transformations of LDP, Varadhan's Integral Lemma, Projections.

February 18 : General Principles - Transformations of LDP, Varadhan's Integral Lemma, Projections (continued).

February 25 : Sample Path LDP - Mogulskii's theorem for Random Walks and Schilder's theorem for Brownian Motion.

March 4 : Sample Path LDP - Friedlin-Wentzell theory for Diffusions with an application to Importance Sampling for Path Dependent Options in Stochastic Volatility Models.

March 11 : Spring Break.

March 18 : General Principles - Abstract Existence Results, LDP in Topological Vector Spaces.

March 25 : Cremár's Theorem in Polish Spaces, Sanov's Theorem on the empirical law of sequences of i.i.d random variables.

April 1 : LDP for the empirical measure of Finite State Markov Chains.

April 8 : LDP for the empirical measure of Markov Chains in Polish Spaces.

April 15 : LDP for the empirical measure of Markov Processes in Polish Spaces.

April 22 : LDP for the empirical measure of Markov Processes in Polish Spaces (continued).

April 29 : Application of occupation measure LDP : Utility Maximization in the limit of a long time horizon.