LECTURE 2 EXERCISES

1 : Show that if $0 \in D_\Lambda^\circ$ then Λ^* is a good rate function.

2 : Let μ be a measure on $\mathbb{R}$ such that $\int_{\mathbb{R}} |x|\mu(dx) < \infty$. Let $\Lambda_\mu, \Lambda_\mu^*$ be the associated cummulant generating function and Legendre transform. Let $p \in \mathbb{R}$.

i) If $\nu = \delta_p * \mu$ is the convolution of μ, δ_p then $\Lambda_\nu^*(x) = \Lambda_\mu^*(x-p)$.

ii) Prove

$$\int_{\mathbb{R}} \exp\left(\alpha \Lambda_\mu^*(x)\right) \mu(dx) \le \frac{2}{1-\alpha} \qquad \forall \alpha \in (0,1)$$

This is from exercises 1.2.9, 1.2.10 in the Deuschel and Stroock book. In ii), let $p = \int_{\mathbb{R}} x\mu(dx)$ and use the facts (and make sure you understand why...)

$$\mu([q,\infty)) \le e^{-\Lambda_\mu^*(q)} \qquad q \ge p$$
$$\mu((-\infty,q]) \le e^{-\Lambda_\mu^*(q)} \qquad q \le p$$

3 : (This is actually a hold-over from Lecture 1) Prove that if $\mathcal{X}$ is a metric space with distance function d and the $\{\mu_n\}$ are Borel measures then $\{\mu_n\}$ cannot satisfy a LDP with more than one rate function (and hence the rate function must be unique).

4 : Let I be a rate function. Let $\delta > 0$ and set $I^\delta(x) = \min\left\{I(x) - \delta, \frac{1}{\delta}\right\}$. Prove that for any set γ

$$\lim_{\delta > 0} \inf_{x \in \Gamma} I^\delta(x) = \inf_{x \in \Gamma} I(x)$$

5 : Do the following exercises from Dembo & Zeitouni.

i) 2.2.23 (page 35)
ii) 2.2.36 (page 41)
iii) 2.2.38 (page 42)