## LECTURE 10 EXERCISES

**1:** Prove that the remark in the middle of page 214 in the Dembo and Zeitouni book is true; i.e. that if $a = \sigma\sigma'$ is uniformly positive definite then $I_x(f) = \hat{I}_x(f)$ where

$$I_x(f) = \inf\left\{\frac{1}{2}\int_0^1 |\dot{g}_t|^2 dt : g \in H_1, f(t) = x + \int_0^t b(f_s)ds + \int_0^t \sigma(f_s)\dot{g}_s ds\right\}$$

$$\hat{I}_x(f) = \begin{cases} \frac{1}{2}\int_0^1 \left|\sigma(f(t))^{-1}\left(\dot{f}_t - b(f_t)\right)\right| dt & f \in H_1^x \\ \infty & \text{else} \end{cases}$$

where

$$H_1^x = \left\{f : f(t) = x + \int_0^t \phi(s)ds, \phi \in L_2([0,1])\right\}$$

**2:** There is nothing to turn in for this problem : Make sure you go through and understand the proof of Theorem 5.6.7 on page 214 in the Dembo and Zeitouni book - especially Lemmas 5.6.9 and 5.6.18.

**3:** Do Exercise 5.6.24 in the Dembo and Zeitouni book.

**Comment :** In Exercise 5.6.24 you do not have to reprove everything : you only need to verify the validity of the results in equations 5.6.11 and 5.6.23 under the assumption that $b$ is Lipschitz but possibly unbounded. However, please write a paragraph explaining why verifying these two facts suffices.