46-944 : Stochastic Calculus for Finance I

Spring 2016, Mini 3

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Course Website: On Blackboard : *www.cmu.edu/blackboard*

Lecture and Examination Schedule: The daytime section meets Tuesdays and Thursdays from 10:30 AM to 12:00 PM in Cooper Auditorium, Tepper School of Business, Pittsburgh. Daytime lectures will not be recorded and transmitted to New York. The evening section meets Wednesdays from 5:30 - 8:30 PM in either Tepper 145 or in New York in NY1 or NY2. Evening lectures will be recorded and transmitted to both New York and Pittsburgh. In order to accommodate a scheduling conflict for Professor Bryant, the lecture on Wednesday, February 17th will instead take place on Thursday February 18th. The lecture will still be from 5:30-8:30 PM in Tepper 145 and broadcast to NY1 or NY2.

The mid-term exam is scheduled for Wednesday, February 3rd, from 7:00 - 8:30 PM. Since all (i.e. both daytime section and evening section) students will take the exam at this time, there will be no daytime section on Thursday, February 4th.

The final exam is scheduled for Monday, February 29th, from 5:30 to 8:30 PM.

Weather permitting, I will be in New York for the lecture on January 20th, for the lecture and mid-term on February 3rd, and for the lecture on February 24th.

Recitation Sections and Discussion Forum: The TA will hold a weekly recitation section on Fridays from 11:00 AM - 12:30 PM in Tepper 145, which will be broadcast to NY1. The purpose of the recitation section is to go over lecture notes, help with homework assignments, and answer general questions relating to the class. Additionally, there is a discussion forum on Blackboard. Questions posted will be answered by either myself or the teaching assistant within 24 hours of posting.

Prerequisites : Probability (46-921), Multi-Period Asset Pricing (46-941).

Primary Textbooks:

> *Stochastic Calculus for Finance II : Continuous-Time Models*
> Steven E. Shreve
> Springer
> ISBN-13: 978-1141923110

> *Stochastic Calculus for Finance I : The Binomial Asset Pricing Model*
> Steven E. Shreve
> Springer
> ISBN-13: 978-0387249681

Canvas Stochastic Calculus Self Study Course: The Stochastic Calculus Self Study (SCSF) course on the Canvas platform will be used as a supplemental learning tool. With regards to our class, the primary use of the SCSF course material is to provide students with an introduction to basic concepts from Probability theory. As such, students are responsible for learning the material in SCSF Modules 1-7. The material in Modules 8 - 19 will be covered in detail throughout the course.

Grading: The course grade is determined as follows:

Midterm Exam : 30%   Final Exam : 60%   Homework : 10%

Exams: Exams are closed book, closed notes, closed calculator, smart phones, smart watches, etc. You may not bring anything but a writing instrument to the exam. Scratch paper will be provided. During the exam, you may not give or receive assistance. Violation of this policy will be treated seriously according to the procedures in the MSCF handbook and may result in expulsion from the program.

Any student who has a course letter grade below B- at the end of the course, and who has at least a 70% homework average, is allowed to take a make-up final. The make-up final will take place during the week of March 6-12. There will be one makeup final exam: students in both the New York and Pittsburgh sections will take the same exam at the same time. The makeup final will not reduce your grade. Additionally, it cannot raise your grade by more than one letter grade, with the maximum grade being a B-. Thus, for example, a student with a C+ before the makeup final may raise her grade to a B-, while a student with a pre-final grade of D may raise his grade to a C.

Homework: There will be 5 homework assignments. Each homework will have 4-5 problems assigned, two of which will be graded. The homework assignments will be posted on or before each Monday morning beginning January 11th. The first four homework assignments are due by 5 PM on the Friday of the week after it was posted, beginning with HW 1 which will be due January 22nd. There is no homework due February 12th because of the exam during the week of February 1-5. The last assignment is due by 5 PM on Thursday February 25th, since February 26th is a reading day.

Homework must be submitted electronically via Blackboard. You are not required to type up your homework assignments, though you are certainly welcome to do so. For handwritten submissions, scan your submission into a .pdf file and then upload the .pdf file. The electronic submission of assignments enables myself and the TA to precisely know when students turned in the assignment, as well as eliminates the possibility of assignments getting lost in the transition to the TA. If, for some reason, you are unable to upload your assignment, please submit your homework in the following ways:

i) In Pittsburgh, give the homework to Jessica Bittel. Please mark "Pittsburgh", as well as your name, on the assignment. Jessica will record the names of students who submit homework.

ii) In New York, give the homework to Seida Muratovic. Please mark "New York", as well as your name, on the assignment. Seida will record the names of students who submit homework.

Late homework may be submitted for a deduction. Late homework must be submitted via Blackboard. Deductions are according to the following schedule:

i) Submission by noon on Saturday following the due date : 20% deduction.

ii) Submission by midnight on Saturday following the due date : 50% deduction.

Homework submissions will not be accepted later than midnight on the Saturday following the due date. Solutions will be posted to Blackboard on the Sunday following the due date. You are encouraged to work with each other on the homework, however, you should not submit any work which you

do not fully understand. In the case where a group of students have suspiciously similar homework submissions, each member of the group may be asked to produce a solution without notes. You may not use course material from previous years to help with the homework : this includes asking students who previously took the course for help.

Course Objectives: This is a first course in stochastic calculus for finance. It revisits the ideas of no-arbitrage pricing and risk-neutral probability measures covered in *Introduction to Fixed-Income* and *Multi-Period Asset Pricing* but now in a continuous time context.

The applications in this course are primarily to the pricing and hedging of equity derivatives. In particular, the Black-Scholes partial differential equation and formula are derived in detail. The stochastic calculus content of the course is also the foundation for fixed income, foreign exchange, commodity and even credit derivatives models.

Topics covered by the course include probability theory in general spaces, with a particular focus on conditioning and change of measure; Brownian Motion; Itô processes and Itô's formula; the Black-Scholes formula; and the Fundamental Theorems of Asset Pricing.

Tentative Course Schedule

Week of January 11: Brownian Motion and Related Topics I

Brownian Motion Definition (Vol II, Sect. 3.3).
Probability Spaces (Vol I, Sect 2.1; Vol II Sect 1.1; SCSF Modules 1,2)
Random Variables, Expectations (Vol I, Sect. 2.2; Vol II, Sect. 1.2, 1.3; SCSF Modules 4,5,6).
Independence (Vol II, Sect. 2.2; SCSF Module 7).
Random Walks and Scaled Random Walks (Vol II, Sect 3.2; SCSF Module 8).
Quadratic Variation of Brownian Motion(Vol II, Sect. 3.4; SCSF Module 9).

Week of January 18: Brownian Motion and Related Topics II

Information, σ-algebras, Filtrations (Vol II, Sect 2.1; SCSF Modules 1,11).
Conditional Expectation (Vol II, Sect. 2.3).
Martingales (Vol I, Sect 2.4).
Martingale Property of Brownian Motion (Vol II, Sect 3.3).
Markov Processes (Vol I, Sect. 2.5).
Markov Property of Brownian Motion (Vol II, Sect 3.5).

Week of January 25: Itô Integrals, Itô 's Formula and the Black-Scholes equation

Itô Integral for Simple Integrands (Vol II, Sect. 4.2).
Itô Integral for General Integrands (Vol II, Sect. 4.3; SCSF Modules 10,12,13).
Itô -Doeblin Formula (Vol II, Sect. 4.4; SCSF Module 14).
Black-Scholes-Merton Formula (Vol II, Sect. 4.5, SCSF Module 17).

Week of February 1: Review and Mid-term Exam

Tuesday February 2: Review day in Pittsburgh. This will be transmitted live to New York and recorded.
Wednesday February 3, 5:30 - 7:00 PM : Review day in New York. I will be in New York and conduct the review. This will be transmitted live to Pittsburgh and recorded.
Wednesday February 3, 7:00 - 8:30 PM : Mid-term Exam for all students.
Thursday February 4 : No Class.

Week of February 8: Multivariate Processes and Risk Neutral Pricing

Multivariate Stochastic Calculus (Vol II, Sect. 4.6).

Lévy Characterization of Brownian Motion (Vol II, Sect. 4.6).
Change of Measure and Girsanov's Theorem (Vol II Sect. 1.6, 5.2).
Risk-Neutral Measure (Vol II, Sect. 5.2).

Week of February 15: Risk Neutral Pricing and the Fundamental Theorems

Martingale Representation Theorem (Vol II, Sect. 5.3).
Derivation of the Risk-Neutral Price for a Contingent Claim (Vol II, Sect. 5.2)
Multi-stock Models, Market Price of Risk Equations (Vol II, Sect 5.2)
Fundamental Theorems of Asset Pricing (Vol II, Sect 5.4).

Week of February 22: Selected Topics and Review

Dividend Paying Stocks (Vol II, Sect. 5.5).
Forwards and Futures (Vol II, Sect. 5.6).
Reflection Principle and Strong Markov Property (Vol II, Sect. 3.7).
Maximum of a Brownian Motion with Drift (Vol II, Sect. 7.2).
Review for Final Exam.

Comprehensive Final Exam : Monday, February 29, 5:30 - 8:30 PM