

21-880 : Stochastic Calculus

Fall 2015

Instructor: Scott Robertson

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Lectures: Monday, Wednesday, Friday: 2 : 30 PM - 3 : 20 PM in Wean Hall 7201.

Prerequisites : Graduate level Measure Theory (21-720) and Probability (21-721). If you have not taken these classes you may still enroll for the course, however, it is expected that students have familiarity with the material taught within these classes. In particular, students should have a working knowledge of Lebesgue integration; L^p spaces; the Radon-Nikodym theorem; conditional expectation; convergence of random variables; and discrete time martingales, with an emphasis on the fundamental inequalities and convergence results.

Primary Textbook:

Brownian Motion and Stochastic Calculus, 2nd edition
Ioannis Karatzas and Steven Shreve
Springer
ISBN-13: 978-0387-97655-6

Additional Textbooks:

Probability with Martingales
David Williams
Cambridge University Press
ISBN-10: 0-521-40605-6

Stochastic Differential Equations
Bernt Øksendal
Springer
ISBN-13: 978-3540-04758-2

Office Hours: Friday 3 : 30 PM - 4 : 50 PM or by appointment.

Grading: The course grade is determined as follows:

Midterm Exam : 30% Final Exam : 30% Homework : 40%

Homework: There will be 6-8 homework assignments (approximately one assignment every two weeks). Homework is due at the beginning of class on the assigned due date. Collaboration is allowed when completing homework assignments, however any submitted work must be essentially your own. Homework submissions which appear suspiciously similar will not be accepted. Late homework will also not be accepted.

Exams: The mid-term exam will take place during the week of October 12 – 16. The final exam will take place during finals week at the end of the semester. Exams are closed notes, closed text-book, etc.

Course Objectives : This class provides a rigorous introduction to the modern theory of stochastic calculus, with a particular emphasis on continuous time, continuous path stochastic processes, the canonical example being Brownian motion. The class will be taught at the graduate level and hence the responsibility for mastering the material is placed squarely on the students. The main topics covered include

- Basic definitions relating to stochastic processes: filtrations; measurability; stopping times; càdlàg processes; Martingales; quadratic variation, etc.
- The fundamental inequalities, convergence results, optional sampling theorem, and other properties of Martingales.
- The Doob-Meyer decomposition; construction of local Martingales; the analysis of the space of continuous square integrable Martingales.
- The definition and construction of Brownian motion: the Kolmogorov-Daniell consistency theorem, the Kolmogorov-Čentsov theorem on continuous modifications; the construction of Wiener measure.
- Analysis of Brownian motion : Markov, strong Markov properties; path regularity and distributional properties.
- The construction of the stochastic integral with respect to a continuous Martingale.
- Itô's change of variable formula; representation results; Girsanov's theorem; local time of Brownian motion.
- Stochastic differential equations and diffusions: construction of strong and weak solutions; the Martingale problem; connections with partial differential equations and harmonic analysis; the Feynman-Kac formula.

Course Outline (tentative!)

- Continuous time Martingales : basic definitions; fundamental inequalities; convergence results; optional sampling theorem; quadratic variation; the Doob-Meyer decomposition; local Martingales; the space of continuous square integrable Martingales.
- Brownian motion : construction via the Kolmogorov-Daniell and Kolmogorov-Čentsov theorems; construction of Wiener measure and the Donsker invariance principle; the Markov and strong Markov property; path regularity and distributional properties.
- Stochastic integration : construction of the stochastic integral with respect to a continuous Martingale; Itô's change of variable formula; Burkholder-Davis-Gundy moment inequalities; representation theorems: Lévy's characterization of Brownian motion, Dambis-Dubins-Schwarz theorem, Brownian Martingale representation.
- Girsanov's theorem and Brownian local time.
- Stochastic differential equations and diffusions: strong and weak solutions with important examples; introduction to diffusions, with an emphasis on the Markov property, Martingale problem, Feynman-Kac formula, Kolmogorov's forward equation, transition densities, and the Dirichlet, Poisson problems.