MATH 690 Methods of Optimization
Fall 2004

1 Course Description

This course provides an introduction to the theory and algorithms for optimization problems with an emphasis on modern computational considerations. The following topics will be covered:

- Convex sets and functions, examples of convex optimization problems
- Duality, Lagrangian function, Lagrangian dual
- Optimality conditions for unconstrained convex optimization, gradient methods, Newton's method, self-concordance
- Linear equality constraints, optimality conditions, solution methods
- Inequality constraints, barrier functions, the central path, interior-point methods
- Nonconvex optimization problems, line-search methods, trust-region methods, sequential quadratic programming

2 Details

Instructor:	Reha Tütüncü, WEH 7219, x8-2558, reha@cmu.edu
Lectures:	MW 3:30-4:50, SH 220
Office Hrs.:	MW 2:00-3:00, WEH 7219
Textbook:	*Convex Optimization* by Boyd and Vandenberghe. `http://www.stanford.edu/~boyd/cvxbook.html`
Grading:	Periodic HWs (30 %), midterm exam (30 %), final exam (40 %).
Prerequisites:	Basic multivariate calculus, some linear algebra.
Homepage:	On BlackBoard

3 Course Outline

I expect to cover most of Chapters 1-5, and 9-11 of the text as well as a few topics not covered in the text. The emphasis will be on the theory of optimization problems and algorithms with special attention to computational issues. There will be 9-10 homeworks some of which will involve coding of algorithms. Here is how I expect the course to proceed:

1. Basics and Theory (13-14 lectures)
    - Optimization models
    - Convex sets and functions
    - Convex Optimization
    - Duality

2. Unconstrained Optimization (5-6 lectures)
    - Optimality conditions
    - Gradient descent and steepest descent methods
    - Newton's method
    - Quasi-Newton methods
3. Equality Constrained Optimization (2-3 lectures)
    - Optimality conditions and Lagrange multipliers
    - Newton's method
4. Inequality Constraints and Interior-Point Methods (5-6 lectures)
    - Barrier function and the central path
    - Barrier method
    - Primal-dual interior-point methods
5. Local Optimization of Nonconvex Problems (2-3 lectures)
    - Line-search and trust-region methods
    - Sequential quadratic programming methods

4 Books on reserve at E&S Library

The following books are (or soon will be) on reserve at the Engineering and Science Library.

1. Convex optimization / Stephen Boyd, Lieven Vandenberghe
2. Numerical optimization / Jorge Nocedal, Stephen Wright
3. Optimization / edited by G.L. Nemhauser, A.H.G. Rinnooy Kan, M.J. Todd
4. Numerical methods for unconstrained optimization and nonlinear equations / J.E. Dennis, Jr., Robert B. Schnabel
5. Practical optimization / Philip E. Gill, Walter Murray, Margaret H. Wright
6. Nonlinear programming / Dimitri P. Bertsekas

5 You should know ...

You should already know, or learn by the end of the first few weeks (reading the Appendix of the textbook or the reference books), the answer to the following questions:

1. How does one solve a system of linear equations? What is Gaussian elimination? What are range and null spaces of matrices?

2. What is a norm? What are the 2-norms, 1-norms, ∞-norms of vectors? What is a matrix norm? What are the eigenvalues of matrices?

3. What is a positive definite matrix? ... positive semi-definite matrix?

4. What are gradient and Hessian of real-valued functions? What is the Jacobian of a vector-valued function?

5. What is the Taylor series expansion of vector valued functions?

6 You will learn ...

Upon completing this course you will be able to answer (most of) the following additional questions:

1. What is an optimization problem? ... a linear/quadratic/nonlinear programming problem? ... a convex optimization problem? ... a least squares problem?

2. What is a convex set? What are some important examples of convex sets?

3. What is a cone? What is a dual cone? What is a generalized inequality?

4. What is the separating hyperplane theorem?

5. What is a convex function? ... a concave function?

6. What is a quasi-convex function?

7. How are the convexity of a function and its derivatives related?

8. What are the optimality conditions for convex optimization problems? What is complementary slackness?

9. What is second-order cone programming? ...geometric programming? ... semidefinite programming?

10. What is the Lagrangian function of a constrained optimization problem? What are Lagrange multipliers? What is the Lagrangian dual function?

11. How is "sensitivity analysis" done for optimization problems?

12. What is a "descent method" for unconstrained optimization? What is gradient descent? What is steepest descent?

13. How does Newton's method work for solution of systems of nonlinear equations? What is its convergence rate? How can Newton's method be used for unconstrained optimization?

14. What are self-concordant functions? What is the convergence behavior of Newton's method for minimizing self-concordant functions?

15. How does one solve convex optimization problems with linear equality constraints? What are the optimality conditions for a linearly constrained optimization problem?

16. How does an optimization algorithm maintain the feasibility of its iterates with respect to the equality constraints?

17. What are null-space matrices? How can they be generated?

18. How can the methods of unconstrained optimization such as Newton's method be generalized to the constrained case?

19. What are interior-point methods for inequality constrained optimization problems?

20. What is the logarithmic barrier function? .. the central path?

21. How does the barrier method work to solve inequality constrained optimization problems? Why is the self-concordance of a barrier function important? What is the computational complexity of interior-point methods for convex optimization problems?

22. What are the optimality conditions for nonconvex optimization problems?

23. What is a line-search method? What is a trust-region method? How can these methods help global convergence of Newton's method for nonconvex optimization problems?

24. What is the sequential quadratic programming method?