21-241 MATRICES AND LINEAR TRANSFORMATIONS
SUMMER I 2012
HOMEWORK 2

(1) Describe the set of solutions to the following system of linear equations, using a particular solution and finitely many solutions to the corresponding homogeneous system. [5]

$$\begin{aligned} 4x - \ 5y + 2z + w &= 1 \\ -y \qquad + w &= 0 \end{aligned}$$

(2) Find the inverse of the following matrix: [10]

$$\begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{3} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \end{pmatrix}$$

(3) Let A and B be square matrices of the same size. Show that if AB is invertible, then so are A and B. [15]

(4) Prove or disprove; there is a (real) 2×2 matrix A such that $A^2 = -I$. [5]

(5) Let $A = \left(\begin{smallmatrix} 1 & 1 \\ 1 & 0 \end{smallmatrix}\right)$. Show by induction on n that

$$A^n = \begin{pmatrix} f_{n+1} & f_n \\ f_n & f_{n-1} \end{pmatrix}$$

where f_n is the nth Fibonacci number, defined by the recursion $f_0 = 0$, $f_1 = 1$, and $f_{n+2} = f_{n+1} + f_n$. [10]

(6) Let $\theta \in [0, 2\pi)$, and let $L \subseteq \mathbb{R}^2$ be the line containing the origin $(0, 0)$ which makes an angle of θ with the x-axis. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the function which takes some point $x \in \mathbb{R}^2$ to its reflection across L.

  (a) Show that T is a linear transformation. (You can use either algebra or geometry. If you use geometry, make sure to be unambiguous when drawing your figures, and to illustrate every possible case.) [10]

  (b) Find the standard matrix of T. [10]