

**21-241 MATRICES AND LINEAR TRANSFORMATIONS
SUMMER I 2012
HOMEWORK 1**

- (1) Solve the following system of equations using Gaussian elimination. [5]

$$x + 2y + 3z = 0$$

$$4x + 5y + 6z = 2$$

$$7x + 8y + 9z = 4$$

You must give a complete description of the set of solutions.

- (2) For which real values of a and b does the following system have a unique solution? or infinitely many solutions? or no solutions? [10]

$$x - 2y = 1$$

$$ax + y = b$$

- (3) Let $X = \{(a, b) \in \mathbb{R}^2 \mid a = b \vee a = -b\}$. Show that there is no system of linear equations whose solution set is exactly X . [10]
- (4) Prove that a swap operation can be implemented by a sequence of row combination and scaling operations. [10]
- (5) Give a complete description of all 2×2 matrices in reduced row echelon form. *Or:* Give a complete description of all 2×2 matrices in row echelon form. [10]
- (6) Prove or disprove: $A^2 = 0$ implies $A = 0$ for all 2×2 matrices A . [15]