MATRICES AND LINEAR TRANSFORMATIONS
21-241 SUMMER 2012
SYLLABUS

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1. Introduction

Matrices and Linear Transformations, 21-241, is an introductory course on what is often called "linear algebra." Linear algebra plays a key role in many areas of pure and applied mathematics, including quantum physics, statistics, analysis, and optimization. Our take on linear algebra will mostly emphasize the theoretical aspects of the subject, including some of what we call abstract algebra. The course will be more of a developing picture than a linear exploration of discrete topics. Many of the concepts we encounter will turn up again and again as we discover more connections between them, and more methods to deal with them.

As with all summer mathematics courses at CMU, the pace will be very fast, and you will have to work hard to keep up. I strongly recommend that you miss as little class time as possible, since we will often cover an entire topic in one or two days. Your work will not only include your homework and exams, but also taking notes at lecture, and going over those notes afterwards to make sure you understand the material.

I will be giving proofs during lecture, and you will be asked to produce proofs too, on your exams as well as your homework. This will be a new experience for many of you, and developing your proof-writing abilities will comprise a lot of the time you spend on the course.

There is a website for the course at

`http://www.math.cmu.edu/~pmckenne/school/summer/12/`

I'll post the homeworks here, as well as the course notes.

2. Prerequisites

There are no official prerequisites for the course, but out of necessity we will sometimes make use of nontrivial topics introduced in 21-127, Concepts of Mathematics. These will include, but are not limited to, the following;

- Basic logic and the notation thereof; quantifiers and connectives.
- Induction.
- Sets and functions; injectivity, surjectivity, bijectivity.
- Equivalence relations.

Each such topic will be given at least a brief introduction before being used.

3. Grading

There will be a homework every week, two midterm exams and a final exam. The grade breakdown is as follows;

40% Homework
20% Exam 1
20% Exam 2
20% Final Exam

I'll assign letter grades based on the standard scale;

A 90-100%
B 80-89%
C 70-79%
D 60-69%
R 0-59%

If necessary I will change this scale at the end of the semester, but only to your benefit. I won't know what I've decided until all the grades are in.

4. Homework

Homeworks will typically be due on Wednesdays, the exception being homework 1 which will be due on Thursday, May 24. They will not be given equal weight; in particular, homework 1 will be short and thus worth less than most of the rest. The homework problems will be worth either 5, 10, 15, or 20 points, and will require either a computation or a proof. The proof problems will usually be worth more than the computational problems. Homework will be due at the beginning of class on the day it's due; late homework will not be accepted.

I strongly suggest that you start working on a homework the day that I make it available (usually the Wednesday that the previous homework was due), since they will typically be far too long for you to complete in just the day (or two days) before the due date. The pace of the course will be fast enough that you often won't have the course material necessary to complete the homework the day it's put out; but it will be structured so that you can work on parts at a time.

5. Exams

There will be three exams including the final; they will be weighted equally. You'll be asked to solve computational problems, recite definitions, and provide some straightforward, and sometimes not so straightforward, proofs. Several but not all of the proof problems will be taken from a list which I will give you at least a week in advance of the exam. This is to motivate you to learn the material necessary for solving the problem (as we're discussing it in class), as well as to give you time to think over your answer.

6. Course policies

- Collaboration on homeworks is encouraged. However, Ill use the so-called whiteboard policy; you are not allowed to write up your solutions during or immediately after discussing them with other students. This is to ensure that, when you do go to write up your solutions, you have to work through the argument yourself. Ill also require that you cite your collaborators in the homework on which you worked.
- Robotic collaboration is also encouraged for help in the more computational problems; but in most cases Ill require that you show your work. The most practical tool to use is Matlab, which is freely available from CMU and already installed on the cluster computers around campus. Also available is Octave, (http://www.gnu.org/software/octave/) the free GNU alternative to Matlab. Octave is installed on the Linux computers around campus and can be easily accessed by running the command octave at a command prompt.
- Robotic collaboration does not include asking the internet for the answer to a homework problem.

7. Course content

The following is a list of some of the topics we'll cover, not necessarily in order.

(1) Systems of linear equations, matrices, and Gaussian elimination.
(2) Invertible matrices.
(3) Subspaces of $\mathbb{R}^n$ and $\mathbb{C}^n$. The null space and range space of a matrix.
(4) Linear independence, spanning sets, and bases.
(5) Dimension.
(6) The trace.
(7) Linear transformations.
(8) Geometry in $\mathbb{R}^n$ and $\mathbb{C}^n$; inner products and orthogonality.
(9) Orthogonal bases; Gram-Schmidt. Orthogonal and unitary transformations.
(10) The determinant.
(11) Eigenvalues and eigenvectors; the characteristic polynomial of a matrix.
(12) Diagonalization of Hermitian matrices.