CNA SEMINARS

FRIDAY, September 1, 2000

CNA/MATH COLLOQUIUM: 4:30 P.M., WeH 7500, *Augusto Visintin*, Universita' degli Studi di Trento, Dipartimento diMatematica, Trento, Italy.
TITLE: "Quasilinear Equations with Hysteresis"
ABSTRACT: Let $\mathcal{F}$ be a hysteresis operator, and A a second order elliptic operator. The following equation arises in elasto-plasticity, ferromagnetism, ferroelectricity:

$$\frac{\partial^2}{\partial t^2}[u + \mathcal{F}(u)] + Au = f. \tag{1}$$

For $\mathcal{F}$ equal to a (possibly discontinuous) scalar Preisach operator, existence of a solution is proved for an associated initial- and boundary-value problem. Processes in ferromagnetic metals and in ferromagnetic insulators can be represented by the following vector equations, respectively,

$$\frac{\partial}{\partial t}[\vec{H} + \vec{\mathcal{F}}(\vec{H}) + \nabla \times \nabla \times \vec{H} = \vec{f}, \tag{2}$$

$$\frac{\partial^2}{\partial t^2}[\vec{H} + \vec{\mathcal{F}}(\vec{H})] + \nabla \times \nabla \times \vec{H} = \vec{f} \tag{3}$$

(here written with normalized coefficients).
Existence of a solution is proved for an initial- and boundary-value problem associated to (2), for $\vec{\mathcal{F}}$ equal to a (possibly discontinuous) vector estension of the Preisach operator.
A similar result is proved for (3), for a smaller class of (discontinuous) hysteresis operators.

TUESDAY, September 5, 2000

CNA SEMINAR: 1:30 P.M., PPB 300, *Paolo Podio-Guidugli*, Univ. di Roma.
TITLE: Strain and Superconductivity
ABSTRACT: TBA

TUESDAY, October 3, 2000

CNA SEMINAR: 1:30 P.M., PPB 300, *Gianpietro DelPiero*, Department of Engineering, University of Ferrara.
TITLE: "On the role of interface energies in the description of material behavior"
ABSTRACT: Research progresses in the subject of material behavior have been very rapid. A common aspect in this development is a tendency to by-passing some of the basic assumptions of classical continuum mechanics. Based on the concept of interface energy, a possibility of a unified view in a broader context, but still at the macroscopic level and within the limits of the continuum scheme, has recently emerged. Indeed, starting from a purely elastic scheme and allowing for the creation of discontinuities at the price of an interface energy, it is possible to obtain a variety of responses, depending on the shape chosen for the function describing the dependence of the interface energy on the amplitude of the discontinuities. This approach has the advantage of using the direct methods of calculus of variations to determine the stable or metastable equilibrium configurations, leaving the way open to traditional techniques for numerical solution. Moreover, it is conceivable to relate the shape chosen for the interface energy to the fine properties of the structure of matter.

TUESDAY, October 10, 2000

CNA SEMINAR: 1:30 P.M., PPB 300, *Arie Leizarowitz*, Technion, Israel.
TITLE: On second order two-dimensional variational problems on unbounded domains.
ABSTRACT: We consider the energy functional

$$J_\Omega = \int_\Omega [(\Delta u)^2 - b|\nabla u|^2 + \psi(u)]dxdy$$

where Ω is a bounded domain in R^2 and ψ is a $C^2(R^2)$ super quadratic potential. Associated with this functional is the notion of mean energy, which is meaningful for a class of configurations u on the whole plane, and we study the corresponding minimization problem. Minimizers of this problem are called *equilibrium configurations*. We establish apriori bounds for this unbounded-domain problem, and use it to establish existence of equilibrium configurations. We also discuss the existence of radially symmetric equilibrium configurations. Moreover, we remark on the connection between radially symmetric configurations and 1-dimensional configurations.

TUESDAY, October 17, 2000

CNA SEMINAR: 1:30 P.M., PPB 300, *A. Pisztora*, Department of Mathematical Sciences, Carnegie Mellon University.
TITLE: "On the equlibrium theory of phase coexistence: a microscopic point of view"
ABSTRACT: The phenomenological theory of coexisting phases assumes the existence of a quantity, called the "surface energy", associated with the phase boundaries. This quantity can be computed as a surface integral of a (direction dependent) scalar quantity (called the surface tension) over the phase boundaries. The theory asserts that in a system in equilibrium the shapes of the phases are determined so as to minimize the total surface energy. (In dimensions more than two the arising variational problems are very hard and remain partly unresolved.)

In this talk I will focus on an other mathematical challange associated with this physical phenomenon: namely, how can we justify the phenomenological picture based on statistical mechanics/probability theory. More precisely I will consider an archetypal microscopic model of a two-component material exhibiting phase coexistence and explain results which imply the aforementioned phenomenological assumptions and answer questions which can't even be addressed remaining on the macroscopic level (such as what the phases are exactly).

I will try to provide all the background necessary to understand the results and will not go into probabilistic details. The talk should be understandable for graduate students.

TUESDAY, October 23, 2000

CNA SEMINAR: 1:30 P.M., PPB 300, *A. Pisztora*, Department of Mathematical Sciences, Carnegie Mellon University.
TITLE: "On the equlibrium theory of phase coexistence: a microscopic point of view, part 2"
ABSTRACT: The phenomenological theory of coexisting phases assumes the existence of a quantity, called the "surface energy", associated with the phase boundaries. This quantity can be computed as a surface integral of a (direction dependent) scalar quantity (called the surface tension) over the phase boundaries. The theory asserts that in a system in equilibrium the shapes of the phases are determined so as to minimize the total surface energy. (In dimensions more than two the arising variational problems are

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October 31, 2000

CNA SEMINAR: 1:30 P.M., PPB 300, *Stuart Hastings*, Dept. of Mathematics, University of Pittsburgh.
TITLE: "Layers, chaos, and global attractors for some one-dimensional reaction diffusion problems"
ABSTRACT: We consider equations of the form $u_t = \epsilon^2 u_{xx} + f(x, u)$ with Neumann boundary conditions. The key early reference is by Angenent, Mallet-Paret and Peletier (1987), who found all stable equilibrium solutions for a class of function $f(x, u)$ which are cubic in u, with $uf(x, u) < 0$ for large u. In joint work with S.B. Ai we study the "Duffing" case $f(x, u) = -u^3 + \lambda u - \cos x$, and find a large class of unstable equilibria, including solutions with multiple internal layers. We also study the ode satisfied by the steady states as a nonlinear oscillator, and obtain some chaos results which do not treat ϵ as a small parameter.

In the case of the Neumann problem for small ϵ we are able to obtain the Morse index of the new solutions. Further, we consider the bifurcation of solutions as a function of λ, and we anticipate that near the initial bifurcation point it will be possible to apply techniques of Fiedler and Rochas to determine the global attractor.

TUESDAY, November 7, 2000

CNA SEMINAR: 1:30 P.M., PPB 300, J. Necas, Charles University at Prague, Czech Republic.
TITLE: "Incompressible fluids in 3d with wiscosity depending on pressure"
ABSTRACT: The incompressible fluid models where the viscosity can de-

pend both on the second invariant of the symmetric velocity gradient and on the pressure were documentated to be important in particular at high pressure. The aim of the lecture is to show that they are physically reasonable viscosity forms, for which the global in time existence of weak solution can be proved. Three-dimensional flows are analyzed.

TUESDAY, November 14, 2000

CNA SEMINAR: 1:30 P.M., PPB 300, *Giovanni P. Galdi*, Department of Mechanical Engineering, University of Pittsburgh.
TITLE: TBA
ABSTRACT: TBA

TUESDAY, December 5, 2000

CNA SEMINAR: 1:30 P.M., PPB 300, *Michal Kowalczyk*, Department of Mathematical Sciences, Carnegie Mellon University.
TITLE: Finite dimensional nonlinear problems and localized solutions to elliptic equations and systems
ABSTRACT: Solving a wide range of semilinear elliptic problems can be reduced to solving a system of nonlinear equations in n real or complex variables (this is sometimes called finite dimensional reduction). Some known examples are elliptic problems involving critical Sobolev exponent, Ginzburg-Landau equation, stationary reaction-diffusion systems and others. Finite dimensional reduction for these equations not only yields the existence of solutions but also allows to describe their important properties such as location of their singularities, vortices or local maxima. Unfortunately solving the finite dimensional problem corresponding to a PDE might be a challenge in itself-for instance showing that the vortices in Ginzburg-Landau equation concentrate along Abrikosov lattice seems to be still an open problem. In my talk I will discuss a reaction-diffusion system in which location of local maxima of one of the components can be obtained by solving a system of nonlinear equations in n complex variables. Exploiting the invariance of the finite dimensional problem with respect to isometries of $\mathbf{C}^n$ and using the Principle of Symmetric Criticality we will be able to establish the existence of interesting patterns such as honeycomb or a snowflake.

TUESDAY, January 30, 2001

CNA SEMINAR: 1:30 P.M., PPB 300, Irene Fonseca, Carnegie Mellon University, Department of Mathematical Sciences.

TITLE: "Micromagnetics : Equilibria and Relaxation

ABSTRACT: Minima of the energy for large magnetic bodies with vanishing induced magnetic fields

$$E(m) := \int_\Omega [\varphi(m) - \langle h_e; m\rangle]\, dx + \frac{1}{2}\int_{\mathbb{R}^3} |h_m|^2\, dx \tag{1}$$

are completely characterized in terms of the anisotropic energy density φ and the applied external magnetic field $h_e \in \mathbb{R}^3$. More generally, one considers an energy functional for a large ferromagnetic body of the form

$$F(m) := \int_{\mathbb{R}^N} f(x, \chi_\Omega(x)m(x), u(x), \nabla u(x))\, dx$$

where $(\chi_\Omega m, \nabla u)$ satisfies Maxwell's equations, i.e. $u \in H^1(\mathbb{R}^N)$ is the unique solution of $\Delta u + \mathrm{div}\,(\chi_\Omega m) = 0$ in $\mathbb{R}^N$. It is shown that if f is a Carathéodory function satisfying very mild growth conditions then the relaxation of F with respect to $L^\infty - w*$ convergence in $L^\infty(\Omega; \overline{B(0,1)})$ is given by

$$\mathcal{F}(m) = \int_\Omega Q_M f(x, m(x), u(x), \nabla u(x))\, dx + \int_{\mathbb{R}^N \setminus \Omega} f(x, 0, u(x), \nabla u(x))\, dx$$

where $Q_M f$ is the quasiconvex envelope of f relative to the underlying partial differential equations. This class of integrands includes those of the type

$$f(x, m, u, h) = \varphi(x, m, u) + \psi(x, u, h)$$

with $\psi(x, u, \cdot)$ non convex.

The first part of this work was undertaken in collaboration with Bernard Dacorogna and the relaxation results were obtained jointly with Giovanni Leoni.

TUESDAY, February 6, 2001

CNA SEMINAR: 1:30 P.M., PPB 300, Morton Gurtin, Carnegie Mellon University Department of Mathematical Sciences.

TITLE: "A gradient theory of single-crystal plasticity that accounts for geometrically necessary dislocations"

ABSTRACT: This talk discusses the kinematics of geometrically necessary dislocations (GNDs) and develops a concomitant gradient theory that accounts for GNDs within a thermomechanical framework. The theory is

based on classical macroforces; microforces for each slip system consistent with a microforce balance; a mechanical version of the second law that includes, via the microforces, work performed during slip; a rate-dependent constitutive theory that includes dependences on a tensorial measure of geometrically necessary dislocations. The microforce balances are equivalent to nonlocal (pde) yield conditions for the individual slip systems. To make contact with classical dislocation theory, the microstresses are shown to represent counterparts of the Peach-Koehler force on a single dislocation.

TUESDAY, February 13, 2001

CNA SEMINAR: 1:30 P.M., PPB 300, *Leonid Berlyand*, , Department of Mathematics and Materials Research Institute, Penn State, University Park, PA.
TITLE: "Network Approximation for Effective Properties of a High Contrast Random Composites"
ABSTRACT: We present a new approach for calculation of effective properties of high contrast random composites (random particles in a matrix or host when properties of the particles and the hosting medium are very different) and provide its rigorous mathematical justification. The main idea of this approach is the reduction of the original continuum problem, which is described by PDE with rough coefficients, to a discrete random network. We introduce the the interparticle distance parameter for nonperiodic (irregular) distributions of particles and show asymptotic equivalence of the effective (homogenized) coefficients for the discrete and continuum models in the limit when the relative interparticle distance goes to zero. Our method is based on variational techniques and it provides an exact error estimate in which all constant are explicitly defined. Such results are rare in homogenization: most of the error estimates provide the order of magnitude only and involve some unknown constants. Our study was motivated by a problem of optimizing effective properties of polymer/ceramic composites. We use the discrete network to compute the effective coefficients numerically and draw physical predictions about polydispersed composites (particles of different sizes) in the percolation regime.

The work was done jointly with A. Kolpakov [1].

1. L. Berlyand, A. Kolpakov. To appear in "Archive of Rational Mecahnics and Analysis".

THURSDAY, February 16, 2001

CNA SEMINAR: 1:30 P.M., PPB 300, *Michel Jabbour*, Department of Mathematical Sciences, Carnegie Mellon University.
TITLE: "On the equations of anisotropic curvature flow in three-dimensions with curvature-dependent energy"
ABSTRACT: In the context of crystal growth or grain boundary motion, the evolution of a phase interface or grain boundary is sometimes kinetically driven, i.e., it is essentially independent of the behavior of the bulk phases. Moreover, for realistic interfacial energies, the anisotropic motion-by-curvature equation exhibits backward-parabolic behavior over intervals of their domain, thus inducing the formation of facets and wrinkles. In this talk, a thermodynamically consistent regularized model is derived, which can be used to analyze such interfacial phenomena. The main ingredient is a constitutive dependence of the interfacial energy density on the curvature tensor which, in our framework, requires the introduction of interfacial configurational moments and yields a generalized Eshelby relation for the surface configurational stress tensor.

TUESDAY, February 20, 2001

CNA SEMINAR: 1:30 P.M., PPB 300, Luc Tartar, Carnegie Mellon University, Department of Mathematical Sciences.
TITLE: "On traces of $W^{2,1}$"
ABSTRACT: The traces on a hyperplane $x_N = 0$ of functions in $W^{1,1}(R^N)$ is exactly $L^1(R^{N-1})$ (Emilio GAGLIARDO, 1957), but the space of traces of functions in $W^{2,1}(R^N)$ is strictly smaller than $W^{1,1}(R^{N-1})$ (Francoise DEMENGEL, 1984). I will describe what I know on the question of characterizing the space of traces.

[Joint work with Francoise DEMENGEL, Departement de Mathematiques, Universite de Cergy-Pontoise, France]

TUESDAY, February 27, 2001

CNA SEMINAR: 1:30 P.M., PPB 300, Alfred Carasso, Mathematical and Computational Sciences Division, National Institute of Standards and Technology, Gaithersburg, MD
TITLE: "Direct Blind Deconvolution of Images and Levy Stable Laws"
ABSTRACT: Blind deconvolution seeks to deblur an image without knowing the cause of the blur. Iterative methods are commonly applied to that problem, but the iterative process is slow, uncertain, and often ill-behaved. This talk considers a significant but limited class of blurs that can be ex-

pressed as convolutions of 2-D radially symmetric Levy 'stable' probability density functions. This class includes Gaussian and Lorentzian (Cauchy) distributions. For such blurs, methods are developed that can detect the point spread function from 1-D Fourier analysis of the blurred image. A separate image deblurring technique uses this detected point spread function to deblur the image. Each of these two steps uses direct non-iterative methods, and requires interactive adjustment of parameters. As a result, blind deblurring of 512X512 images can be accomplished in minutes of CPU time on current desktop workstations. Numerous blind experiments on synthetic data show that for a given blurred image, several distinct point spread functions may be detected that lead to useful, yet visually distinct reconstructions. Application to real blurred images will also be demonstrated.

TUESDAY, March 6, 2001

CNA SEMINAR: 1:30 P.M., PPB 300, *M. Foss*, Carnegie Mellon University, Department of Mathematical Sciences.
TITLE: "Lavrentiev's Phenomenon in Nonlinear Elasticity"
ABSTRACT: Lavrentiev's phenomenon is associated with the sensitivity of a variational problem's infimum to the regularity required of the competing mappings. We present one of a class of physically natural stored energy densities where the energy functional exhibits the Lavrentiev phenomenon.

TUESDAY, March 20, 2001

CNA SEMINAR: 1:30 P.M., PPB 300, *Martin Feinberg*, Department of Chemical Engineering, Ohio State University.
TITLE: "Lecture 1. Chemical Engineering for Mathematicians"
ABSTRACT: This first lecture is intended to provide background for the second. Although a great deal of hard-core chemical engineering is mathematically complex, the questions themselves are fairly easy to understand. I'll try to describe, in mathematical terms, how chemical engineers think about systems of chemical reactions and how those reactions enter into processes designed to enhance the production of good things and suppress the production of bad things. At the very least, it should become clear how the occurrence of chemical reactions gives rise to systems of nonlinear equations. With luck, it will also become clear why – in light of the great freedom one has in designing chemical processes – questions of process design are mathematically rich.

THURSDAY, March 22, 2001

CNA SEMINAR: 1:30 P.M., PPB 300, *Martin Feinberg*, Department of Chemical Engineering, Ohio State University.
TITLE: "Lecture 2. Toward a Theory of Process Design"
ABSTRACT: Given a network of chemical reactions and given a set of reactant streams, one wants to know how best to utilize those streams in order to enhance the production rate of a desired product while suppressing the production rates of unwanted by-products. The set of qualitative design strategies is so vast that it becomes difficult to know not only which design outcome is optimal but also which outcomes are even feasible - that is, which outcomes are attainable by some design and which are not. There is beginning to emerge a theory for assessing outer limits on what can be attained from all possible designs, even those that remain unimagined. The theory brings together parts of chemical engineering and parts of mathematics (e.g, convexity theory, dynamical systems, differential geometry) in surprising ways. I'll try to survey a little of what's known and some of what's not.

TUESDAY, April 3, 2001

CNA SEMINAR: 1:30 P.M., PPB 300, *Gilles Clermont*, University of Pittsburgh.
TITLE:
ABTRACT:

THURSDAY, April 19, 2001

CNA SEMINAR: 1:30 P.M., PPB 300, Giovanni Leoni,
TITLE: TBA
ABSTRACT: TBA

TUESDAY, April 24, 2001
CNA SEMINAR: 1:30 P.M., PPB 300, Olga Ladyzhenskaya, Steklov Instiute of Mathematics, St. Petersburg, Russia.
TITLE: TBA
ABSTRACT: TBA

THURSDAY, April 26, 2001

CNA SEMINAR: 1:30 P.M., PPB 300, Anna Vainchtein, University of Pittsburgh, Department of Mathematics.

TITLE: TBA
ABSTRACT: TBA