Homework Set 4

1) Suppose that Eventown has fewer than $2^{\lfloor n/2 \rfloor}$ clubs. Prove that there is room for a new club without violating the Eventown rules.

2) Show that if n is even, then there exist at least $2^{n(n+2)/8}/(n!)^2$ nonisomorphic solutions to the Oddtown problem of size n. Prove that for large n this is greater than $2^{n^2/9}$.

3) Let V be a vector space of dimension n over K. Let V^{**} be the dual space of V^*. Give an explicit isomorphism between V and V^{**}.

4) Let V be finite dimensional over R with positive definite scalar product. Let A be an operator on V. Show that the image of A^T is the orthogonal space to the kernel of A.