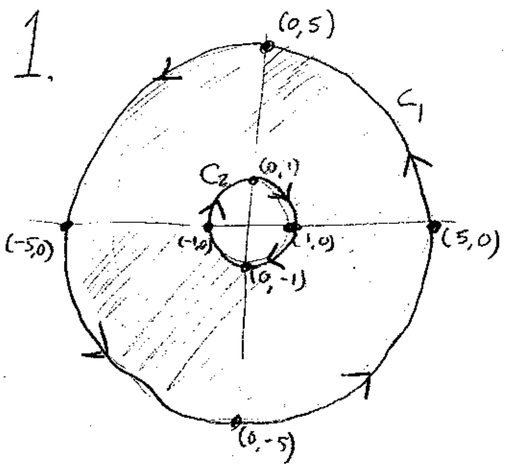
Green's Theorem

We will calculate the same thing 3 times and (hopefully) get the same thing all three times.

1.



(a) draw arrows on C_1 and C_2 so that the shaded region is on the left as you follow the arrows.

(b) Parameterize C_1

$r_1(t) = \langle 5\cos t, 5\sin t \rangle$

$0 \le t \le 2\pi$

(c) Parameterize C_2

$r_2(t) = \langle \cos t, -\sin t \rangle$

$0 \le t \le 2\pi$

(d) check that your parameterizations follow the curves in the direction you intend them to. in r_2, sin is negative.

(e) calculate dx and dy for C_1

$dx = -5\sin t$

$dy = 5\cos t$

(f) calculate dx and dy for C_2

$dx = -\sin t$

$dy = -\cos t$

(g) set up the line integral $\int_{C_1+C_2} xy^2\,dx + x\,dy$ in terms of t.

$$\int_0^{2\pi} -625\cos t\sin^3 t\,dt + \int_0^{2\pi} 25\cos^2 t\,dt + \int_0^{2\pi} -\cos t\sin^3 t\,dt + \int_0^{2\pi} -\cos^2 t$$

(h) evaluate the integral.

(h)

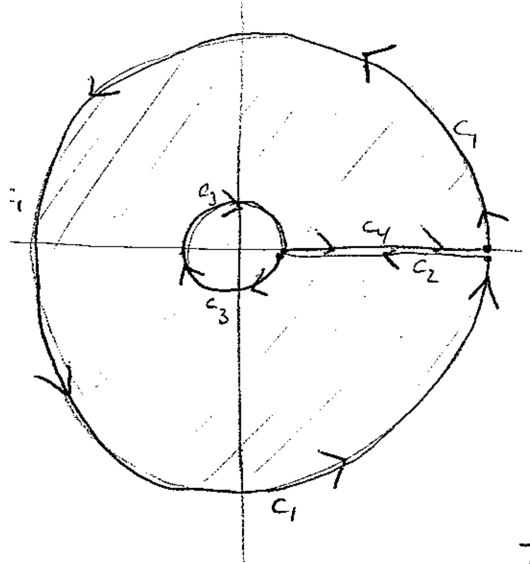
$$\int_0^{2\pi} -625\cos t\,\sin^3 t\,dt + 25\int_0^{2\pi}\frac{1+\cos 2t}{2}\,dt + \int_0^{2\pi} -\cos t\,\sin^3 t\,dt - \int_0^{2\pi}\frac{1+\cos 2t}{2}\,dt$$

$$= \left[-625\,\frac{\sin^4 t}{t}\right]_0^{2\pi} + 25\left[\frac{t}{2} + \frac{\sin 2t}{2}\right]_0^{2\pi} + \left[-\frac{\sin^4 t}{4}\right]_0^{2\pi} - \left[\frac{t}{2} + \frac{\sin 2t}{4}\right]_0^{2\pi}$$

$$= 0 + 25(\pi) + 0 - (\pi)$$

$$= \boxed{24\pi}$$

2.



(a) draw arrows on C_1, C_2, C_3, C_4 so that the shaded region is on the left as you follow the arrows.

(b) write out $\int_C xy^2\,dx + x\,dy$ as four integrals (don't worry about "t"'s just yet)

$$\int_{C_1} \vec{F}\cdot dA + \int_{C_2} \vec{F}\cdot dA + \int_{C_3} \vec{F}\cdot dA + \int_{C_4} \vec{F}\cdot dA$$

(c) what is the relationship between C_2 and C_4?

Negatives of each other

(d) using this information, rewrite $\int xy^2\,dx + x\,dy$

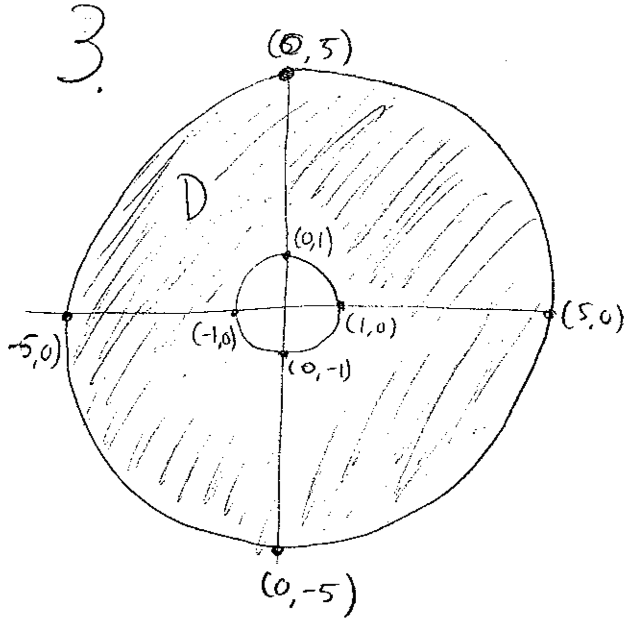
the second and fourth integral cancel.

$$\int_{C_1} xy^2\,dx + x\,dy + \int_{C_3} xy^2\,dx + x\,dy$$

(e) Have you done this integral before? what is the answer?

This is exactly number 1, so the answer is 24π

3.



(a) in the previous problem, what were P and Q?

$P = xy^2$

$Q = x$

(b) what is $Q_x - P_y$?

$1 - 2xy$

(c) we are going to integrate $\iint_D (Q_x - P_y)\,dA$. what coordinate System suits D the best?

polar.

(d) Set up the integral

$$\int_{\theta=0}^{2\pi}\int_{r=1}^{5} (1-2r^2\cos\theta\sin\theta)\, r\, dr\, d\theta$$

(e) evaluate the integral.

$$\int_0^{2\pi}\int_1^5 (r-2r^3\cos\theta\sin\theta)\,dr\,d\theta$$

$$= \int_0^{2\pi}\left[\frac{r^2}{2} - \frac{r^4}{2}\cos\theta\sin\theta\right]_{r=1}^{5} d\theta = \int_0^{2\pi} (12 - 312\cos\theta\sin\theta)\,d\theta = \left[12\theta - 312\,\frac{\sin^2\theta}{2}\right]_0^{2\pi}$$

$$= 12(2\pi) = 24\pi$$

(f) is your answer the same as in number 1 & 2?

yes. Acording to Green's Theorem, these should be the same, and they are.