Change of Variables

Concept: When the region we integrate over doesn't lend itself to regtangular or polar coordinates, make up new variables, u and v.
Computation: The boundarys will often be four sided. Choose u to be two of the sides, and v to be the other two. Then use $\frac{\partial(x,y)}{\partial(u,v)} = \text{abs}\begin{vmatrix} x_u & x_v \\ y_u & y_v \end{vmatrix}$ to find the extra factor.
Evaluate the following integrals

1. $\iint_R (x - 3y)dA$, where R is the triangular region with vertices (0,0), (2,1), and (1,2)
2. $\iint_R (x - 3y)dA$, where R is the quadralateral with vertices (0,0), (2,2), (1,-2), and (3,0)
3. $\iint_R xydA$, where R is the region in the first quadrent bounded by the lines $y = x$ and $y = 3x$ and the hyperbolas $xy = 1$ and $xy = 3$.
4. $\iint_R \frac{x - 2y}{3x - y}dA$, where R is enclosed by the lines $x - 2y = 0$, $x - 2y = 4$, $3x - y = 1$, and $3x - y = 8$.
5. (hard) $\iint_R \cos\left(\frac{y - x}{y + x}\right) dA$, where R is the polygon with vertices (1,0), (2,0), (0,2), and (0,1).
6. (hard) $\iint_R e^{x+y}dA$, where R is given by the inequality $|x| + |y| \leq 1$

Answers

1. -3
2. 15
3. $2\ln 3$
4. $\frac{8}{5}\ln 8$
5. $\frac{3}{2}\sin 1$
6. $e - e^{-1}$