ANSWERS 3/29

Integrals

(29) $\int \frac{e^x+1}{e^x}\,dx = \int (1+e^{-x})\,dx$ $\qquad \boxed{\begin{aligned} u&=-x \\ du&=-dx \end{aligned}}$

$= -\int (1+e^u)\,du = -u - e^u + c = \boxed{x - e^{-x} + c}$

(43) $\int_{x=1}^{2} x\sqrt{x-1}\,dx = \int_{u=0}^{1} (u+1)u^{1/2}\,du = \int_{u=0}^{1} u^{3/2} + u^{1/2}\,du = \left[\frac{2}{5}u^{5/2} + \frac{2}{3}u^{3/2}\right]_0^1 = \frac{2}{5} + \frac{2}{3} = \boxed{\frac{16}{15}}$

$x-1=u$
$x=u+1$
$dx=du$

(3) $\int x\cdot\cos 5x\,dx = x\cdot\frac{1}{5}\sin 5x - \int \frac{1}{5}\sin 5x\,dx = \boxed{\frac{x}{5}\sin 5x + \frac{1}{25}\cos 5x + c}$

$u=x \qquad v=\frac{1}{5}\sin 5x$
$du=dx \qquad dv=\cos 5x\,dy$

(17) $\int_1^2 \frac{\ln x}{x^2}\,dx = \left[-\frac{\ln x}{x}\right]_1^2 - \int_1^2 -\frac{1}{x^2}\,dx = -\frac{\ln 2}{2} + \frac{\ln 1}{1} - \left[\frac{1}{x}\right]_1^2 = -\frac{\ln 2}{2} - \frac{1}{2} + 1$

$= \boxed{\frac{1}{2} - \frac{\ln 2}{2}}$

$u=\ln x \qquad v=-x^{-1}$
$du=\frac{1}{x}dx \qquad dv=x^{-2}dx$

(9) $\int \frac{x-9}{(x+5)(x-2)}\,dx = \int \left(\frac{2}{x+5} + \frac{-1}{x-2}\right)dx = \boxed{2\ln(x+5) - \ln(x-2) + c}$

(19) $\int \frac{1}{(x+5)^2(x-1)}\,dx = \int \underset{A}{\frac{1/36}{x-1}} + \underset{B}{\frac{-17/180}{x+5}} + \underset{C}{\frac{-1/6}{x+5^2}} = \boxed{\frac{1}{36}\ln(x-1) - \frac{17}{180}\ln(x+5) + \frac{1/6}{x+5} + c}$

$1 = A(x+5)^2 + B(x+5)(x-1) + C(x-1)$ $\qquad B = \frac{1}{5}\left(\frac{25}{36} - 1 - \frac{1}{6}\right) = -\frac{17}{36\cdot 5}$

$1 = 25A - 5B - C$

Mixed Integrals

1. $u = \arctan 4t \quad v = t$

$du = \frac{4}{1+(4t)^2} \quad dv = dt$

$t\arctan 4t - \int \frac{4t}{1+16t^2}\,dt = \boxed{t \cdot \arctan 4t - \frac{1}{8}\ln(1+16t^2) + c}$

2. $\int \frac{x-1}{(x+1)(x+2)}\,dx = \int \frac{-2}{x+1} + \frac{3}{x+2}\,dx = \boxed{-2\ln(x+1) + 3\ln(x+2) + c}$

3. $u = 2x+3$

$du = 2dx \qquad 3\int u^{-3}\,du = -\frac{3}{2}u^{-2} + c = \boxed{-\frac{3}{2}(2x+3)^{-2} + c}$

4. $u = x \quad v = e^x$

$du = dx \quad dv = e^x$

$xe^x - \int e^x\,dx = \boxed{xe^x - e^x + c}$

5. $u = \sin t$

$du = \cos t \qquad \int u^5\,du = \frac{1}{6}u^6 + c = \boxed{\frac{1}{6}\sin^6 t + c}$

6. $\int \frac{x+3}{(x-2)(x-4)}\,dx = \int \frac{-\frac{5}{2}}{x-2} + \frac{\frac{7}{2}}{x-4}\,dx = \boxed{-\frac{5}{2}\ln(x-2) + \frac{7}{2}\ln(x-4) + c}$

Double Integrals

1. $$\int_1^2\int_{-1}^1 xy\,dx\,dy = \int_1^2 \left[\frac{x^2}{2}\cdot y\right]_{x=-1}^{1} dy = \int_1^2 y-y\,dy = 0.$$

2. $$\int_0^1\int_{1/2}^2 \frac{x}{y}\,dy\,dx = \int_0^1 \left[x\ln y\right]_{y=\frac{1}{2}}^{2} dx = \int_0^1 x\ln 2 - x\ln\tfrac{1}{2}\,dx = \int_0^1 2x\ln 2\,dx$$

$$= \left[x^2\ln 2\right]_{x=0}^{1} = \boxed{\ln 2}$$

3. $$\int_1^5\int_0^\pi xy\cdot\sin x\,dx\,dy = \int_1^5\left(\left[-xy\cos x\right]_0^\pi + \int_0^\pi y\cos x\,dx\right)dy$$

$u = x \quad v = -\cos x$

$du = dx \quad dv = \sin x\,dx$

$$= \int_1^5 -\pi y(-1) + 0 + \left[y\cdot\sin x\right]_{x=0}^{\pi}\,dy = \int_1^5 \pi y\,dy = \left[\frac{\pi y^2}{2}\right]_1^5 = \frac{25\pi-\pi}{2} = \boxed{12\pi}$$

4. $$\int_0^1\int_0^{x^2} x+2y\,dy\,dx = \int_0^1 \left[xy+y^2\right]_{y=0}^{x^2} dx = \int_0^1 x^3+x^4 = \left[\frac{x^4}{4}+\frac{x^5}{5}\right]_{x=0}^{1} = \frac{9}{20}$$

(5) $$\int_0^1 \int_y^{e^y} \sqrt{x}\,dx\,dy = \int_0^1 \left[\frac{2}{3}x^{3/2}\right]_{x=y}^{x=e^y} dy = \int_0^1 \frac{2}{3}e^{\frac{3y}{2}} - \frac{2}{3}y^{\frac{3}{2}}\;dy$$

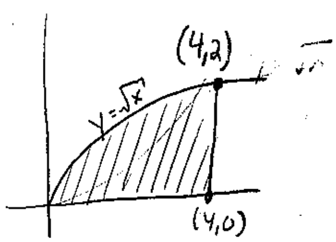
$$= \left[\frac{4}{9}e^{\frac{3y}{2}} - \frac{4}{15}y^{\frac{5}{2}}\right]_0^1 = \frac{4}{9}e^{3/2} - \frac{4}{15} + \frac{4}{9} - 0$$

$$= \boxed{\frac{4}{9}e^{3/2} + \frac{8}{45}}$$

(6) $$\int_0^{\pi/2}\int_0^{\cos\theta} e^{\sin\theta}\,dr\,d\theta = \int_0^{\pi/2}\left[re^{\sin\theta}\right]_{r=0}^{\cos\theta} d\theta = \int_0^{\pi/2} \cos\theta\, e^{\sin\theta}\,d\theta$$

$$= \left[e^{\sin\theta}\right]_{\theta=0}^{\theta=\pi/2} = e^1 - e^0 = \boxed{e-1}$$

(7)



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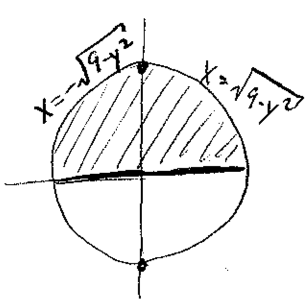

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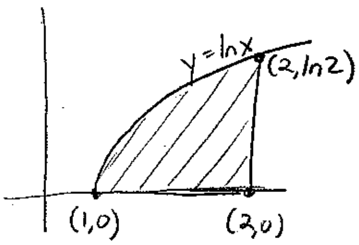

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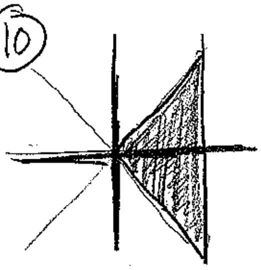
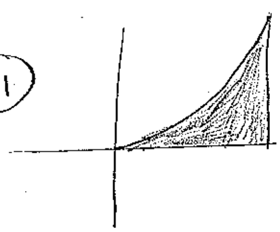


$$\int_0^2\int_{-x}^{x} x^3y^2\,dy\,dx = \int_0^2 \left[\frac{x^3y^3}{3}\right]_{y=-x}^{x} dx = \int_0^2 \frac{2x^6}{3}\,dx = \left[\frac{2}{21}x^7\right]_0^2$$

$$= \boxed{\frac{256}{21}}$$

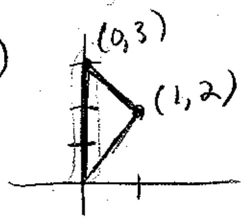
(11)



$$\int_0^1\int_0^{x^2} x\cdot\cos y\,dy\,dx = \int_0^1 \left[x\sin y\right]_{y=0}^{x^2} dx$$

$$= \int_0^1 x\sin x^2\,dx = \left[-\frac{1}{2}\cos x^2\right]_0^1 = \boxed{-\frac{1}{2}\cos 1 + \frac{1}{2}}$$

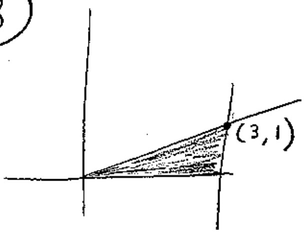
(12)



$$\int_0^1\int_{2x}^{3-x} 2xy\,dy\,dx = \int_0^1 \left[xy^2\right]_{y=2x}^{3-x} dx = \int_0^1 x(x-3)^2 - 4x^3\,dx$$

$$= \int_0^1 x^3 - 6x^2 + 9x - 4x^3\,dx = \left[\frac{-3}{4}x^4 - 2x^3 + \frac{9}{2}x^2\right]_0^1 = -\frac{3}{4} - 2 + \frac{9}{2} = \boxed{-\frac{1}{4}}$$

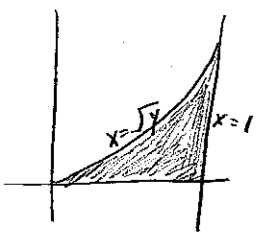
(13)



$$\int_0^1\int_{y=0}^{y=\frac{x}{3}} e^{x^2}\,dy\,dx = \int_0^1 \left[ye^{x^2}\right]_{y=0}^{y=\frac{x}{3}} dx = \int_0^1 \frac{x}{3}e^{x^2}\,dx$$

$$= \left[\frac{1}{6}\, e^{x^2}\right]_0^1 = \boxed{\frac{1}{6}(e-1)}$$

(14)



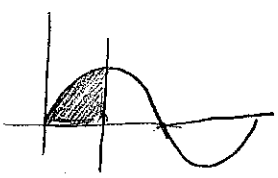
$$\int_{x=0}^1\int_{y=0}^{y=x^2} \sqrt{x^3+1}\,dy\,dx = \int_0^1 \left[y\sqrt{x^3+1}\right]_{y=0}^{y=x^2} = \int_0^1 x^2\sqrt{x^3+1}\,dx$$

$$= \left[\frac{1}{3}\frac{2}{3}(x^3+1)^{3/2}\right]_0^1 = \boxed{\frac{2}{9}\left(2^{3/2}-1\right)}$$

(15)

$x = \arcsin y$

$\sin x = y$



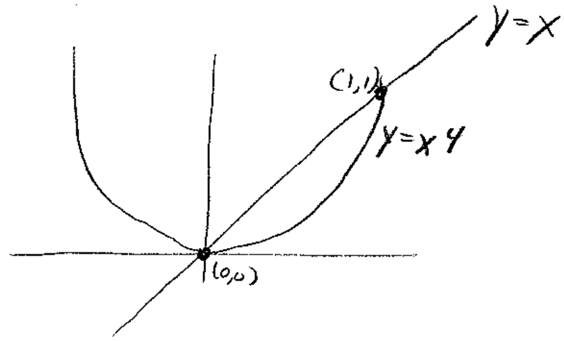
$$\int_0^{\frac{\pi}{2}}\int_{y=0}^{y=\sin x} \cos x\sqrt{1+\cos^2 x}\,dy\,dx = \int_0^{\frac{\pi}{2}} \left[y\cos x\sqrt{1+\cos^2 x}\right]_{y=0}^{\sin x} dx$$

$$= \int_0^{\frac{\pi}{2}} \sin x\cdot\cos x\cdot\sqrt{1+\cos^2 x}\,dx = \left[\frac{1}{2}\,\frac{2}{3}\left(1+\cos^2 x\right)^{3/2}\right]_{x=0}^{\frac{\pi}{2}}$$

$$= \frac{1}{3}(1+0)^{3/2} - \frac{1}{3}(1+1)^{3/2} = \boxed{\frac{1}{3}\left(1-2^{3/2}\right)}$$

(16) $z = 2y + x \leftarrow$ Function

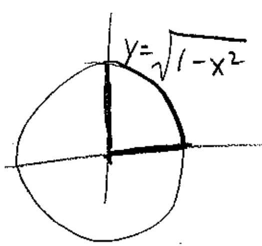
$\left.\begin{array}{l} y = x \\ y = x^4 \end{array}\right\}$ domain



$$\int_0^1 \int_{y=x^4}^{x} (2y+x)\, dy\,dx = \int_0^1 \left[y^2 + xy\right]_{y=x^4}^{y=x} dx = \int_0^1 2x^2 - x^8 - x^5\, dx$$

$$= \left[\frac{2}{3}x^3 - \frac{x^9}{9} - \frac{x^6}{6}\right]_0^1 = \frac{2}{3} - \frac{1}{9} - \frac{1}{6} = \boxed{\frac{7}{18}}$$

(17)



$$\int_0^1 \int_{y=0}^{\sqrt{1-x^2}} y\, dy\,dx = \int_0^1 \left[\frac{y^2}{2}\right]_{y=0}^{y=\sqrt{1-x^2}} dx$$

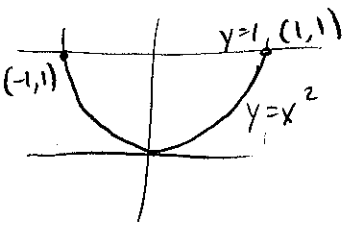
$$= \int_0^1 \frac{1-x^2}{2}\, dx = \frac{1}{2}\left[x - \frac{x^3}{3}\right]_0^1 = \frac{1}{2}\left(1 - \frac{1}{3}\right) = \boxed{\frac{1}{3}}$$

(18) planes intersect at

$3y = 2+y$

$y = 1$

So this is our top Curve.



$$\int_{x=-1}^{x=1} \int_{y=x^2}^{y=1} (2+y) - (3y)\, dy\,dx$$

$$= \int_{-1}^{1} \int_{x^2}^{1} 2 - 2y\, dy\,dx = \int_{-1}^{1} \left[2y - y^2\right]_{x^2}^{1} dx$$

$$= \int_{-1}^{1} 1 - 2x^2 + x^4\, dx = \left[x - \frac{2}{3}x^3 + \frac{1}{5}x^5\right]_{-1}^{1}$$

$$= 2\left(1 - \frac{2}{3} + \frac{1}{5}\right) = \frac{16}{15}$$