ANSWERS 2/24

1. $f_x = 2xy + y^2$ $\qquad$ $f_y = x^2 + 2xy$

   $f_{xx} = 2y$ $\qquad$ $f_{yx} = 2x + 2y$

   $f_{xy} = 2x + 2y$ $\qquad$ $f_{yy} = 2y$

2. $f_x = -(y + \tan y)e^{-x}$ $\qquad$ $f_y = (1 + \sec^2 y)e^{-x}$

   $f_{xx} = (y + \tan y)e^{-x}$ $\qquad$ $f_{yx} = -(1 + \sec^2 y)e^{-x}$

   $f_{xy} = -(1 + \sec^2 y)e^{-x}$ $\qquad$ $f_{yy} = (2 \cdot \sec y \cdot \sec y \cdot \tan y)e^{-x}$

3. $f_x = \frac{y}{xy} = \frac{1}{x}$ $\qquad$ $f_y = \frac{x}{xy} = \frac{1}{y}$

   $f_{xx} = -\frac{1}{x^2}$ $\qquad$ $f_{yx} = 0$

   $f_{xy} = 0$ $\qquad$ $f_{yy} = -\frac{1}{y^2}$

4. $f_x = -\sin(x)\sin(y)$ $\qquad$ $f_y = \cos(x)\cos(y)$

   $f_{xx} = -\cos(x)\sin(y)$ $\qquad$ $f_{yx} = -\sin(x)\cos(y)$

   $f_{xy} = -\sin(x)\cos(y)$ $\qquad$ $f_{yy} = -\cos(x)\sin(y)$.

5. $f_x(x,y) = \cos(xy) - xy\sin(xy)$

   $f_x(3,\pi) = \cos(3\pi) - 3\pi\sin(3\pi) = \boxed{-1}$

6. $f_y(x,y) = \frac{x^2}{2\sqrt{x+y}}$

   $f_y(2,7) = \frac{4}{2\sqrt{9}} = \boxed{\frac{2}{3}}$

7. $f_x(x,y) = 0$

   $f_x(2,1) = 0$.

8. $f_y(x,y) = x^y \cdot \ln(x)$

$f_y(3,2) = 3^2 \cdot \ln(3) = \boxed{9 \cdot \ln(3)}$

9. $f_x(x,y) = y\, x^{y-1}$

$f_x(3,2) = 2 \cdot 3^1 = \boxed{6}$

10. $\frac{\partial}{\partial x}(x^2+y^2) = \frac{\partial}{\partial x} \sin(yz)$

$2x = \cos(yz) \cdot y \cdot \frac{\partial z}{\partial x}$ $\qquad \frac{\partial z}{\partial x} = \frac{2x}{y\cos(yz)}$

$\frac{\partial}{\partial y}(x^2+y^2) = \frac{\partial}{\partial y} \sin(yz)$

$2y = \cos(yz) \cdot \left(z + y\frac{\partial z}{\partial y}\right)$ $\qquad \frac{\partial z}{\partial y} = \frac{\frac{2y}{\cos(yz)} - z}{y}$

11. $e^y = y\frac{\partial z}{\partial x} \Rightarrow \boxed{\frac{1}{y}e^y = \frac{\partial z}{\partial x}}$

$xe^y = \frac{\partial z}{\partial y} \cdot y + z \Rightarrow \boxed{\frac{1}{y}(xe^y - z) = \frac{\partial z}{\partial y}}$

12. $\frac{\partial z}{\partial x} + 1 = y\left(z + x\frac{\partial z}{\partial x}\right)$

$\frac{\partial z}{\partial x} - xy\frac{\partial z}{\partial x} = yz - 1 \Rightarrow \boxed{\frac{\partial z}{\partial x} = \frac{yz-1}{1-xy}}$

$\frac{\partial z}{\partial y} = x\left(z + y\frac{\partial z}{\partial y}\right)$

$\frac{\partial z}{\partial y} - xy\frac{\partial z}{\partial y} = xz \Rightarrow \boxed{\frac{\partial z}{\partial y} = \frac{xz}{1-xy}}$

13. $4 + \cos(z)\dfrac{\partial z}{\partial x} = y^2 \dfrac{\partial z}{\partial x}$

$4 = \dfrac{\partial z}{\partial x}\left(y^2 - \cos z\right) \implies \dfrac{\partial z}{\partial x} = \dfrac{4}{y^2 - \cos z}$.

$\cos(z)\dfrac{\partial z}{\partial y} = 2yz + y^2 \dfrac{\partial z}{\partial y}$

$\dfrac{\partial z}{\partial y}(\cos z - y^2) = 2yz \implies \dfrac{\partial z}{\partial y} = \dfrac{2yz}{\cos z - y^2}$

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14. $f_x = 6x = 6$

$f_y = 2y = 4$

$\boxed{z = 6(x-1) + 4(y-2) + 7}$

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16. $f_x = e^y = 1$

$f_y = xe^y = 1$

$\boxed{z = 1(x-1) + 1(y-0) + 1}$

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15. $f_x = \cos x \cos y = 1$

$f_y = -\sin x \sin y = 0$

$z = 1(x - \pi) + 1(y - \pi)$

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17. $f_x = \dfrac{\sqrt{y}}{x} = \dfrac{4}{e}$

$f_y = \dfrac{\ln x}{2\sqrt{y}} = \dfrac{1}{4}$

$z = \dfrac{4}{e}(x - e) + \dfrac{1}{4}(y - 4) + 2$

18. $f_x = -\frac{1}{2} y x^{-3/2} + \frac{1}{2} x^{-1/2}$ $\qquad f_y = \frac{1}{2} x^{-1/2} + 1$

$f = y x^{-1/2} + x^{1/2} + g_1(y)$ $\qquad f = y x^{-1/2} + y + g_2(x)$

$$\Rightarrow \boxed{f = \frac{y}{\sqrt{x}} + \sqrt{x} + y + C}$$

19. $f_x \Rightarrow f = \cos(xy) + x \sin y + g_1(y)$

$f_y \Rightarrow f = (x+1)\sin y + \cos(xy) + g_2(x)$

$= x \sin y + \sin y + \cos(xy) + g_2(x)$

$$\boxed{f = x \sin y + \cos(xy) + \sin y + C}$$

20. $f_x \Rightarrow \quad f = \ln x + \frac{1}{xy} + g_1(y)$

$f_y \Rightarrow \quad f = \frac{1}{y^2} + \frac{1}{xy} + g_2(x)$

$$\boxed{f(x,y) = \frac{1}{xy} + \frac{1}{y^2} + \ln x + C}$$

21. $f_x \Rightarrow \quad f(x,y) = x \sin^2 y + g_1(y)$

$f_y \Rightarrow \quad f(x,y) = -\frac{1}{2}(x+1)\cos(2y) = -\frac{1}{2}(x+1)\left(\cos^2 y - \sin^2 y\right) + g_2(x)$

$= -\frac{1}{2}(x+1)\left(1 - 2\sin^2 y\right) + g_2(x)$

$= -\frac{1}{2}x + x\sin^2 y + \sin^2 y - \frac{1}{2} + g_2(x)$

$$\boxed{f = x \sin^2 y + \sin^2 y + C}$$

$g_1(y) = \sin^2 y, \quad g_2(x) = \frac{1}{2}x + \frac{1}{2}$