Contour Plots

Concept: Draw pictures of surfaces without having to plot in 3 dimensions
Computation: Fix a value of $f(x, y)$, plot the curve, and label it with the fixed value of z.

Sketch the contour plots of the following surfaces

1. $f(x, y) = (y - 2x)^2$
2. $f(x, y) = y - \ln x$
3. $f(x, y) = ye^x$
4. $f(x, y) = \sqrt{y^2 - x^2}$

Limits

Concept: Finding the value of a function "near" a particular point.
Computation: To show that a limit exists, use the squeeze theorem. To show that a limit doesn't exist, find two paths that approach the point, but that give different limits.

Find the limit, if it exists, or show that the limit does not exist.

1. $\lim_{(x,y)\to(5,-2)} x^5 + 4x^3y^2$
2. $\lim_{(x,y)\to(0,0)} \frac{y^4}{x^4 + 3y^4}$
3. $\lim_{(x,y)\to(0,0)} \frac{xy\cos y}{3x^2 + y^2}$
4. $\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2 + y^2}}$
5. $\lim_{(x,y)\to(0,0)} \frac{2x^2y}{x^4 + y^2}$

Partial Derivatives

Concept: The slope of a surface when traveling in the x or y direction.
Computation: To take $\frac{\partial f}{\partial x}$ $(= f_x)$, take a derivative as you normally would wrt x, treating y as a constant. Similar for $\frac{\partial f}{\partial y}$ $(= f_y)$.

Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$

6. $z = x^2 y + xy^2$
7. $z = \dfrac{y + \tan(y)}{e^x}$
8. $z = \ln(xy)$
9. $z = \cos(x)\sin(y)$
10. $z = x\cos(xy)$
11. $z = x^2\sqrt{x + y}$
12. $z = \sin(y^3)$