CALC IN 3D ANSWERS 2/15

1. $s(t) = \int_0^t |\vec{r}'|\, du$

$\vec{r}'(u) = \langle -1, 4, 6 \rangle$,

$|\vec{r}'(u)| = 9$

$\int_0^t 9\,du = [9u]_{u=0}^{t} = 9t.$

$s = 9t \Rightarrow t = \frac{s}{9}$

$r(s) = \langle 1 - \frac{s}{9},\ \frac{4}{9}s + 3,\ \frac{6}{9}s \rangle.$

2. $|\vec{r}'(u)| = |\langle 5\cos t,\ -3\sin t,\ -4\sin t \rangle| = \sqrt{25\cos^2 t + 9\sin^2 t + 16\sin^2 t} = 5$

$s = \int_0^t 5\,du = [5u]_0^t = 5t$

$r(s) = \langle 5\sin\frac{s}{5},\ 3\cos\frac{s}{5},\ 4\cos\frac{s}{5} \rangle.$

3. $|\vec{r}'(u)| = |\langle \frac{d}{dt}(\ln t+1)^3, 0, 0 \rangle = \frac{d}{dt}(\ln t + 1)^3$

$s = \int_0^t \frac{d}{du}(\ln u + 1)^3 = \left[(\ln u + 1)^3\right]_0^t = (\ln t + 1)^3$

$r(s) = \langle s, 6, 7 \rangle.$

4. $f'(t) = \langle 2\cos t,\ 5,\ -2\sin t \rangle$

$|f'(t)| = \sqrt{4+25} = \sqrt{29}$

$\hat{T} = \frac{1}{\sqrt{29}} \langle 2\cos t,\ 5,\ -2\sin t \rangle$

$\hat{T}' = \frac{1}{\sqrt{29}} \langle -2\sin t,\ 0,\ -2\cos t \rangle$

$|\hat{T}'| = \frac{1}{\sqrt{29}} \sqrt{4\sin^2 t + 4\cos^2 t} = \frac{2}{\sqrt{29}}$

$K(t) = \dfrac{2/\sqrt{29}}{\sqrt{29}} = \dfrac{2}{29}$

5. $f'(t) = \langle 2t,\ \cos t - \cos t + t\sin t,\ -\sin t + \sin t + t\cos t \rangle$

$= \langle 2t,\ t\sin t,\ t\cos t \rangle$

$|f'(t)| = \sqrt{4t^2 + t^2} = t\sqrt{5}$

$\hat{T} = \frac{1}{\sqrt{5}} \langle 2,\ \sin t,\ \cos t \rangle$

$\hat{T}' = \frac{1}{\sqrt{5}} \langle 0,\ \cos t,\ -\sin t \rangle$

$|\hat{T}'| = \frac{1}{\sqrt{5}} \cdot 1$

$K(t) = \dfrac{1/\sqrt{5}}{t\sqrt{5}} = \dfrac{1}{t}$

6. $\vec{r}'(t) = \langle \sqrt{2}, e^t, -e^{-t} \rangle$

$|\vec{r}'(t)| = \sqrt{2 + e^{2t} + e^{-2t}}$

$= \sqrt{(e^t + e^{-t})^2} = e^t + e^{-t}$

$\hat{T} = \dfrac{1}{e^t + e^{-t}} \langle \sqrt{2}, e^t, -e^{-t} \rangle$

quotient rule.

$\hat{T}' = \dfrac{\langle 0, e^t, e^{-t} \rangle (e^t + e^{-t}) - (e^t - e^{-t}) \langle \sqrt{2}, e^t, e^{-t} \rangle}{(e^t + e^{-t})^2}$

$= \langle -e^t + e^{-t}, 2, 2e^{-2t} \rangle$

$|\hat{T}'| = \sqrt{e^{2t} - 2 + e^{-2t} + 4 + 4e^{-4t}} = \sqrt{e^{2t} + 2 + e^{-2t} + 4e^{-4t}}$

$K(t) = \dfrac{|\hat{T}'|}{|\vec{r}'|} = \sqrt{\dfrac{e^{2t} + \cancel{2} + e^{-2t} + 4e^{-4t}}{2 + e^{2t} + e^{-2t}}}$

7. $\vec{r}' = \langle 2t, 0, 1 \rangle$

$\vec{r}'' = \langle 2, 0, 0 \rangle$

$|r' \times r''| = \left| \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2t & 0 & 1 \\ 2 & 0 & 0 \end{vmatrix} \right| = |2\hat{j}| = 2$

$|r'| = \sqrt{1 + 4t^2}$

$K = \dfrac{|r' \times r''|}{|r'|^3} = \dfrac{2}{(1 + 4t^2)^{3/2}}$

8. $\vec{r}\,' = \langle 1, 1, 2t \rangle$

$\vec{r}\,'' = \langle 0, 0, 2 \rangle$

$$|\vec{r}\,' \times \vec{r}\,''| = \left| \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 2t \\ 0 & 0 & 2 \end{vmatrix} \right| = \left| \hat{i}(2) - \hat{j}(2) + \hat{k}(0) \right| = 2\sqrt{2}$$

$$|\vec{r}\,'| = \sqrt{2+4t^2}$$

$$K(t) = \frac{2\sqrt{2}}{(2+4t^2)^{3/2}}$$

9. (a) highest curvature at the vertex, lowest at ∞

(b) 7 (and also 8).

(c) The curvature is highest when the denominator is lowest. This occurs at $t=0$.

(d) The vertex in problem 7 is at the origin, when $t=0$, so yes, this is what we expected.