Logic and Definitions

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Warm Up

1. (ACOPS) Prove that if a is rational and b is irrational, then $a+b$ is irrational.

Injective Functions

1. (C.J.) Rigorously define what it means for a function to be injective/one-to-one.
2. (Misha) Let $f : A \to B$ and $g : B \to C$ be functions, and define $h : A \to C$ by $h = g \circ f$. Prove that if h is injective, then f is injective. Is the converse true?
3. (C.J.) Let $f : A \to B$ be a function. Prove that f is injective if and only if there is a function $g : B \to A$ such that $g(f(x)) = x$ for all $x \in A$.

Problems: Affine Planes

Let P be a nonempty set of 'points' and let L be a nonempty set of 'lines' where a line is a subset of P. Two lines are *parallel* if they contain no common points. The pair (P, L) is an *affine plane* if

(a) For every distinct $p_1, p_2 \in P$, there is a unique line $l \in L$ such that $p_1, p_2 \in l$.

(b) For every line l and point p not contained in l, there is a unique line l' that contains p and is parallel to l.

To exclude trivial cases, we will also assume that an affine plane has at least three points and no line contains every point. Let (P, L) be an affine plane, and do the following:

4. Prove that if l_1, l_2, l_3 are distinct lines such that l_1 is parallel to l_2 and l_2 is parallel to l_3, then l_1 is parallel to l_3.
5. Give an example of a affine plane with finitely many points.
6. Prove that every line contains at least two points.
7. Prove that in a finite affine plane, every line contains the same number of points.
8. (Bonus) Construct as many affine planes as you can!

Homework: A Game

Bill and Will play the following game. Initially, there is a stone on each lattice point (x, y) with x, y between 1 and 10. Bill plays first, and the players alternate turns. On each player's turn, he chooses a point (x, y) that still has a stone on it and removes that stone, and all stones down and to the left of that stone. That is, he removes the stone from each (x', y') such that $x' \leq x$ and $y' \leq y$ (that still has a stone). The player who takes the last stone loses.

1. Define rigorously what a 'strategy' in this game is.
2. Define as rigorously as you can the statement "Player X has a winning strategy."
3. Who has a winning strategy in this game? Prove your answer.