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[illegible] #1

1) $\begin{cases} 5x+8y=34 \\ 12x+4y=36 \end{cases}$

$24x+8y=72$

$5x+8y=34$

$19x=38$

$\underline{x=2}$ $\quad 5(2)+8y=34 \Rightarrow \underline{y=3}$

2/2

This linear system comes from a shopping trip to Entropy. x is the price of apples and y is the price of oranges. When I buy 5 apples and 8 oranges, my total is \$34. When I buy 12 apples and 4 oranges, my total is \$36.

The solution is $\left\{\begin{pmatrix} 2 \\ 3 \end{pmatrix}\right\}$ and represents that the cost of an apple is \$2 and the cost of an orange is \$3.

2) (a) $\begin{cases} x-y=0 \\ 2x+y=3 \end{cases} \Rightarrow 3x=3 \Rightarrow \underline{x=1}$

$\Downarrow$

$(1)-y=0 \Rightarrow \underline{y=1}$

$\boxed{\begin{pmatrix} 1 \\ 1 \end{pmatrix}}$

(b) $\begin{cases} x+5y=-1 \\ -x+y=-5 \\ 2x+4y=4 \end{cases} \Rightarrow 6y=-6 \Rightarrow \underline{y=-1}$

$\Downarrow$

$x+5(-1)=-1 \Rightarrow \underline{x=4}$

$\boxed{\begin{pmatrix} 4 \\ -1 \end{pmatrix}}$

(c) $x_1=t \quad x_2=1+t \quad x_3=2-t$

$x_2=1+x_1 \quad x_3=2-x_1$

$-x_1+x_2=1 \quad x_1+x_3=2$

$$\boxed{\begin{cases} -x_1+x_2+0=1 \\ x_1+0+x_3=2 \end{cases}}$$

Now let $x_3=s$.

$x_1+s=2 \Rightarrow \underline{x_1=2-s}$

$-(2-s)+x_2=1 \Rightarrow x_2=\underline{3-s}$

$$\boxed{\begin{pmatrix} 2-s \\ 3-s \\ s \end{pmatrix}}$$

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Let a = spades, b = hearts, c = clubs, d = diamonds

The system can be written:

$$\left[\begin{array}{cccc|c} 2 & 0 & 0 & 1 & 9 \\ 1 & 1 & 0 & 2 & 9 \\ 0 & 2 & 1 & 1 & 9 \end{array}\right] \xrightarrow[R_1 \leftrightarrow R_2]{} \left[\begin{array}{cccc|c} 1 & 1 & 0 & 2 & 9 \\ 2 & 0 & 0 & 1 & 9 \\ 0 & 2 & 1 & 1 & 9 \end{array}\right] \xrightarrow[R_2 - 2R_1]{} \left[\begin{array}{cccc|c} 1 & 1 & 0 & 2 & 9 \\ 0 & -2 & 0 & -3 & -9 \\ 0 & 2 & 1 & 1 & 9 \end{array}\right] \xrightarrow{-\frac{1}{2}R_2} \left[\begin{array}{cccc|c} 1 & 1 & 0 & 2 & 9 \\ 0 & 1 & 0 & \frac{3}{2} & \frac{9}{2} \\ 0 & 2 & 1 & 1 & 9 \end{array}\right]$$

$$\xrightarrow[R_3 - 2R_2]{} \left[\begin{array}{cccc|c} 1 & 1 & 0 & 2 & 9 \\ 0 & 1 & 0 & \frac{3}{2} & \frac{9}{2} \\ 0 & 0 & 1 & -2 & 0 \end{array}\right] \xrightarrow[R_1 - R_2]{} \left[\begin{array}{cccc|c} 1 & 0 & 0 & \frac{1}{2} & \frac{9}{2} \\ 0 & 1 & 0 & \frac{3}{2} & \frac{9}{2} \\ 0 & 0 & 1 & -2 & 0 \end{array}\right]$$

a, b, c are leading variables
d is a free variable

call $\underline{d = t}$

$R_1 \Rightarrow a + \frac{1}{2}t = \frac{9}{2} \Rightarrow \underline{a = \frac{9-t}{2}}$

$R_2 \Rightarrow b + \frac{3}{2}t = \frac{9}{2} \Rightarrow \underline{b = \frac{9-3t}{2}}$

$R_3 \Rightarrow c + 2t = 0 \Rightarrow \underline{c = 2t}$

so the solution set is: $\begin{pmatrix} \frac{9-t}{2} \\ \frac{9-3t}{2} \\ 2t \\ t \end{pmatrix}$

to find how many solutions are non-negative integers, find for what values of t all a, b, c, d are non-negative integers.

$\frac{9-t}{2} \geq 0 \Rightarrow t \leq 9$

$\frac{9-3t}{2} \geq 0 \Rightarrow t \leq 3$

$2t \geq 0 \Rightarrow t \geq 0$

$t \geq 0 \Rightarrow t \geq 0$

these are all true when $\underline{0 \leq t \leq 3}$

Now I will check $t=0, t=1, t=2, t=3$

when $t=0$, $a = \frac{9-0}{2}$ so $t=0$ does not work.

when $t=1$, $a=4$, $b=3$, $c=2$, $d=1$ so $t=1$ works. ✓

when $t=2$, $a = \frac{9-2}{2}$ so $t=2$ does not work.

when $t=3$, $a=3$, $b=0$, $c=6$, $d=3$ so $t=3$ works. ✓

Thus, $\boxed{\text{2 solutions}}$ are composed of non-negative integers.

$$\left\{ \begin{pmatrix} 4 \\ 3 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ 0 \\ 6 \\ 3 \end{pmatrix} \right\}$$ ✓

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