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21-880: Advanced Stochastic Calculus-I
Fall 2008

General Information and Course Description

Instructor	Kavita Ramanan
Office	Wean Hall 6204
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Office hours	Tuesdays 3:30-4:30
Time and venue	MW 3:30-4:50, Wean Hall 5304
Course website	http://www.math.cmu.edu/users/kramanan/teaching/SCALC1/scalc1.html

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Objective. Advanced Stochastic Calculus-I is an introduction to the theory of continuous time stochastic processes, with a special emphasis on Brownian motion. The broad goals of the course are to introduce the theory of stochastic integration, Itô's formula and the tools of stochastic calculus, and to provide some insight into its connection with applications.

Prerequisites. Basic measure theory, probability and real analysis, including knowledge of the weak and strong laws of large numbers and central limit theorems. Knowledge of the basic theory of discrete parameter martingales would be highly useful.

Syllabus. The course will cover the following topics.

1. Basic definitions of stochastic processes, including measurability, filtrations and stopping times;
2. Brownian motion – motivation for its study, definition, constructions, sample path properties, stochastic properties;
3. Properties of continuous martingales, the Doob Meyer decomposition and the quadratic variation process, semimartingales;
4. Markov processes, the strong Markov property and applications;
5. Stochastic integration – motivation, construction, properties of the integral;
6. Itô's formula and its applications;
7. Reflected Brownian motion (local time and Tanaka's formula, if time permits)

Text book. *Introduction to Stochastic Integration* by Hui-Hsiung Kuo
(We will cover Chapters 1–8 and a few select topics from Chapters 10 and 11)
Additional handouts and supplementary reading will be provided when required.

Reference Books On Reserve (at the E&S Library, 4th floor, Wean Hall)

1. Probability, Leo Breiman (LB)
   (This is for revision of background material required for the course, though it contains more advanced material as well)

2. Probability, Davar Khoshnevisan (DK)
   (This is for revision of background material required for the course, though it also contains a bit about Brownian motion at the end)

3. Continuous Martingales and Brownian Motion, Revuz and Yor (RY)

4. Stochastic Differential Equations, Bernt Oksendal (BO)

5. Copies of important handouts will also be placed on reserve at the E&S library.

Exams and Grading.

- Mid-term exam (Monday, October 13th) - 35 %
- Final exam (Monday, December 1st) – 45 %
- Class presentations (November 24 and December 3rd) – 10 %
- Homeworks – 10 %
  (The homework grade will be based on random problems graded from submitted homeworks)

Homeworks and Assignments. Homework assignments will be announced during lectures (with corresponding due dates), and posted on the website.
In addition, each student in the class will present a paper related to stochastic calculus (a discussion of presentations will begin after the mid-term, in the third week of October).
Discussions between students on the course material is encouraged. Discussions of assignments is also permitted, but students are expected to write up their own homeworks and excessive help taken from someone else should be explicitly acknowledged.

Attendance. Regular attendance and completion of homeworks is the key to success in this course. Students are encouraged to participate actively in class and regularly contact the instructor during office hours to clear doubts in a timely fashion.
