Why Learn About Brownian Motion ?

Brownian motion is mathematically a very interesting process with very intriguing properties. In addition, Brownian motion and processes derived from it, such as the Brownian bridge, the Bessel process, reflected Brownian motion, etc., arise in a variety of contexts. For instance, they can be used to provide insight into the following questions.

1. (**Gambling**) Assuming so-called "proportional betting", how does your expected win rate in a game of blackjack depend on the number of decks used and the number of cards dealt before reshuffling ? What is the relationship between the agressiveness of betting and risk, which is measured, say, as the probability of your wealth dropping below a certain specified level before exceeding another specified amount?

2. (**Finance**) Suppose the price X_t of your house is varying randomly with time, and that there is a fixed transaction cost a that will be charged at the time that you sell your house. If you decide to sell your house at time τ, the discounted net of your sale is

$$e^{-\rho\tau}(X_\tau - a),$$

where $\rho > 0$ is a given discount factor. What is the optimal time to sell your house so as to maximize your discounted net ?

3. (**Operations Research**) Consider a wireless system in which you have packets arriving at an average rate of $\lambda > 0$ to a buffer with finite capacity b and they are transmitted on a first-in-first-out basis at a rate μ that depends on the power level chosen, which could be dynamically varied based on system status. Packets that arrive when the buffer is full are lost. Suppose that given a rate of power level θ, the energy consumption up to time t is given by

$$\int_0^t c(\theta(Z(s))\, ds$$

where Z is the state of the buffer. How should you choose $\theta(\cdot)$ to minimize the long-run average energy consumption:

$$\limsup_{t\to\infty} E\left[\frac{1}{t}\int_0^t c(\theta(Z(s)))\, ds\right],$$

where c is an increasing function, subject to an upper bound $\hat{\beta}$ on the long-run average packet drop rate ?

4. (**Combinatorics**) Let $G(n, n+k)$ denote a simple labelled connected graph (i.e., a graph with labelled vertices, undirected edges and without self-loops or multiple edges) with n vertices and $n+k$ edges, and let $c(n, n+k)$ denote the number of connected graphs on n vertices and $n+k$ edges, and let $\tilde{c}(n, n+k)$ denote the number of connected graphs on n vertices and $n+k$ edges that are triangle-free. For a fixed k, what is the relation between $c(n, n+k)$ and $\tilde{c}(n, n+k)$ as $n \to \infty$?

5. (**Connections with Analysis, Statistical Physics ...**)