21-127 Practice Problems

1. (i) Use the Euclidean Algorithm to compute the greatest common divisor of 212 and 188, and then find all integer solutions of $212x + 188y = \gcd(212, 188)$. Show your work.

(ii) Apples cost 50 cents apiece and bananas cost 30 cents apiece. Joe spends a total of 40 dollars on apples and bananas, eats an apple and then finds that he can split the remaining apples into 5 equal groups and the bananas into 6 equal groups. How many bananas did Joe buy? Show your work.

2. (i) Define a relation Ω on the integers by $(a, b) \in \Omega \iff a\,b \equiv 1 \mod 5$. Is Ω an equivalence relation? Explain why or why not.

(ii) Define a relation Γ on the integers by $(a, b) \in \Gamma \iff 5 \mid (a - b)^2$. Is Γ an equivalence relation? Explain why or why not.

3. Show that if a and b are coprime integers, then the greatest common divisor of $2a^2$ and b^2 must be 1 or 2.

4. Let x be an odd integer. Find, with proof, $\gcd(2x^4,\ x^2 + 1)$.

5. Define a relation Ω on $\mathbb{Z} \times \mathbb{Z}$ by $(a, b)\,\Omega\,(c, d) \iff a + d \equiv b + c \mod 3$. Prove that Ω is an equivalence relation.

6. Show that $1^2 - 2^2 + 3^2 - 4^2 + \cdots + (-1)^{n-1}n^2 = (-1)^{n-1}[n(n + 1)/2]$ for every natural number n.

7. Let x be a real number. Show that if $x + 1/x$ is an integer, then $x^n + 1/x^n$ is an integer for every positive integer n.