1 What Is This Sheet?

If you've never seen any counting before, it can be pretty strange and very tricky. The idea here is to try to build up some intuition by looking at problems (and their solutions once I write them). As you examine these, don't worry too much about writing formal proofs. Just try to get a solid understanding of how to arrive at each count.

Also, I should credit Professor Po-Shen Loh for his role in the creation of this sheet. I got the ideas for the vast majority of these problems from him.

2 Problems

1. How many ways are there for Alice, Bob, Charlie, and Denise to stand in a line if Alice must be somewhere to the left of Bob in the line? (Note: When the numbers are small, you can always just list out the cases and then develop faster ways to arrive at the answer). (Ideas: permutations; use of division).

2. How many ways are there for Alice, Bob, Charlie, Denise, and Evan to stand in a line if Alice must be somewhere to the left of Bob and Charlie must be somewhere to the left of Denise? (Similar ideas to the last one).

3. How many ways are there for the same 5 people from above to stand in line if Alice and Bob must be next to each other? What if they must not be next to each other?

4. How many anagrams of the word MISSISSIPPI are distinct? (Ideas: permutations).

5. You have 10 beads in a line. You plan to paint each one either red, green, or blue. How many ways can you paint your beads if at least one of them must be red? What if we instead require that there must be an adjacent pair of beads with the same color (Ideas: complementary counting).

6. You have 6 beads in a line. You plan to color each one either red or blue. How many ways can you do this if 2 red beads cannot be next to each other (a blue bead can be next to anything)? (Ideas: recursions).

7. In a group of people, 15 have been to Argentina, 10 to Bangladesh, and 6 to China. Some of these counts overlap: 5 have been to both Argentina and Bangladesh, 2 to both Bangladesh and China, 3 to both China and Argentina, and 1 person to all three countries. How many people in this group have been to at least one of the three countries? (Hint: try drawing a Venn Diagram). (Ideas: Inclusion-Exclusion).

8. In a 4×4 grid of points, how many ways can we choose 4 points such that the 4 chosen points form a rectangle whose bottom edge is parallel to the line connecting the bottom left and bottom right corners of the grid?

9. In a 3×3 grid of points, how many ways can we choose 3 points such that the three points form a triangle? (Ideas: complementary counting).

10. How many triples of integers (a, b, c) satisfy $0 \leq a \leq b \leq c \leq 6$? (Bijection and distribution).

11. How many permutations of *abcde* leave none of the letters in their original positions? (Inclusion-exclusion).
12. How many sequences of 5 digits (each integer in the sequence is 0-9) are strictly increasing? How many sequences of 5 digits are non-decreasing?
13. How many 3-digit locker combinations (where each digit is 0-9) have the property that no two adjacent digits are equal to each other and the sum of the digits is even?
14. Prove that every set of 10 distinct numbers between 1 and 100 contains 2 disjoint nonempty subsets with the same sum. (Pigeonhole).
15. How many sequences of 10 numbers that contain each of the digits 0-9 exactly once have the property that each number other than the first is within 1 of some term that precedes it?
16. How many sequences of length 6 where each digit is an integer from 1-5 contain an even number of even numbers?

3 Solutions

(Currently under construction. Coming soon!)