

1, Diestel 7.17: Prove the Erdős-Sós conjecture for the case when the tree considered is a path. (Hint. Use Exercise 9 of Chapter 1.)

2, Diestel 7.18: Can large average degree force the chromatic number up if we exclude some tree as an induced subgraph? More precisely: For which trees T is there a function $f : \mathbb{N} \rightarrow \mathbb{N}$ such that, for every $k \in \mathbb{N}$, every graph of average degree at least $f(k)$ either has chromatic number at least k or contains an induced copy of T ?

3, Diestel 7.21: Given a graph G with $\varepsilon(G) \geq k \in \mathbb{N}$, find a minor $H \preceq G$ such that $\delta(H) \geq k \geq |H|/2$.

4, Diestel 7.24: Show that any function h as in Lemma 3.5.1 satisfies the inequality $h(r) > \frac{1}{8}r^2$ for all even r , and hence that Theorem 7.2.3 is best possible up to the value of the constant c .

5, Diestel 7.33: Prove Hadwiger's conjecture for $r = 4$ from first principles. Hint: Use induction on $|G|$. Color a shortest cycle in G (if G has a cycle) and extend the coloring to the rest of the graph.

6, Diestel 7.35: Prove Corollary 7.3.5: A graph with $n > 2$ vertices and no K^5 minor has at most $3n - 6$ edges.
