6. $\left\{-\frac{1}{4}, \frac{2}{9}, -\frac{3}{16}, \frac{4}{25}, \ldots\right\}$. The numerator of the nth term is n and its denominator is $(n+1)^2$. Including the alternating signs, we get $a_n = (-1)^n \dfrac{n}{(n+1)^2}$.

12. $a_n = \dfrac{\sqrt{n}}{1+\sqrt{n}} = \dfrac{1}{1/\sqrt{n}+1}$, so $a_n \to \dfrac{1}{0+1} = 1$ as $n \to \infty$. Converges

26. $\lim\limits_{x\to\infty} \dfrac{(\ln x)^2}{x} \overset{\text{H}}{=} \lim\limits_{x\to\infty} \dfrac{2(\ln x)(1/x)}{1} = 2 \lim\limits_{x\to\infty} \dfrac{\ln x}{x} \overset{\text{H}}{=} 2 \lim\limits_{x\to\infty} \dfrac{1/x}{1} = 0$, so by Theorem 3, $\lim\limits_{n\to\infty} \dfrac{(\ln n)^2}{n} = 0$. Convergent

34. $a_n = \dfrac{2n-3}{3n+4}$ defines an increasing sequence since for $f(x) = \dfrac{2x-3}{3x+4}$,

$f'(x) = \dfrac{(3x+4)(2)-(2x-3)(3)}{(3x+4)^2} = \dfrac{17}{(3x+4)^2} > 0$. The sequence is bounded since $a_n \geq a_1 = -\frac{1}{7}$ for $n \geq 1$, and

$a_n < \dfrac{2n-3}{3n} < \dfrac{2n}{3n} = \dfrac{2}{3}$ for $n \geq 1$.

40. We use induction. Let P_n be the statement that $0 < a_{n+1} \leq a_n \leq 2$. Clearly P_1 is true, since $a_2 = 1/(3-2) = 1$. Now assume that P_n is true. Then $a_{n+1} \leq a_n \Rightarrow -a_{n+1} \geq -a_n \Rightarrow 3 - a_{n+1} \geq 3 - a_n \Rightarrow$

$a_{n+2} = \dfrac{1}{3-a_{n+1}} \leq \dfrac{1}{3-a_n} = a_{n+1}$. Also $a_{n+2} > 0$ (since $3 - a_{n+1}$ is positive) and $a_{n+1} \leq 2$ by the induction hypothesis, so P_{n+1} is true. To find the limit, we use the fact that $\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} a_{n+1} \Rightarrow L = \frac{1}{3-L} \Rightarrow$

$L^2 - 3L + 1 = 0 \Rightarrow L = \frac{3\pm\sqrt{5}}{2}$. But $L \leq 2$, so we must have $L = \frac{3-\sqrt{5}}{2}$.

6. $\sum\limits_{n=1}^{\infty} \dfrac{(-6)^{n-1}}{5^{n-1}}$ is a geometric series with $a = 1$ and $r = -\frac{6}{5}$. The series diverges since $|r| = \frac{6}{5} > 1$.

8. $\sum\limits_{n=0}^{\infty} \dfrac{1}{\left(\sqrt{2}\right)^n}$ is a geometric series with ratio $r = \dfrac{1}{\sqrt{2}}$. Since $|r| = \dfrac{1}{\sqrt{2}} < 1$, the series converges. Its sum is

$$\frac{1}{1-1/\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{2}-1} = \frac{\sqrt{2}}{\sqrt{2}-1} \cdot \frac{\sqrt{2}+1}{\sqrt{2}+1} = \sqrt{2}\left(\sqrt{2}+1\right) = 2+\sqrt{2}.$$

10. $\sum\limits_{n=1}^{\infty} \dfrac{n+1}{2n-3}$ diverges since $\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} \dfrac{n+1}{2n-3} = \dfrac{1}{2} \neq 0$. [Use (7), the Test for Divergence.]

16. $\sum\limits_{n=1}^{\infty} \left[(0.8)^{n-1} - (0.3)^n\right] = \sum\limits_{n=1}^{\infty} (0.8)^{n-1} - \sum\limits_{n=1}^{\infty} (0.3)^n$ [difference of two convergent geometric series]

$$= \frac{1}{1-0.8} - \frac{0.3}{1-0.3} = 5 - \frac{3}{7} = \frac{32}{7}$$

22. For the series $\sum_{n=1}^{\infty} \ln \frac{n}{n+1}$,

$$s_n = (\ln 1 - \ln 2) + (\ln 2 - \ln 3) + (\ln 3 - \ln 4) + \cdots + [\ln n - \ln(n+1)]$$
$$= \ln 1 - \ln(n+1) = -\ln(n+1) \quad \text{[telescoping series]}$$

Thus, $\lim_{n\to\infty} s_n = -\infty$, so the series is divergent.

26. $6.2\overline{54} = 6.2 + \frac{54}{10^3} + \frac{54}{10^5} + \cdots = 6.2 + \frac{54/10^3}{1 - 1/10^2} = \frac{62}{10} + \frac{54}{990} = \frac{6192}{990} = \frac{344}{55}$

28. $\sum_{n=0}^{\infty} 2^n(x+1)^n = \sum_{n=0}^{\infty} [2(x+1)]^n = \sum_{n=1}^{\infty} [2(x+1)]^{n-1}$ is a geometric series with $r = 2(x+1)$, so the series converges $\Leftrightarrow$ $|r| < 1$ $\Leftrightarrow$ $|2(x+1)| < 1$ $\Leftrightarrow$ $|x+1| < \frac{1}{2}$ $\Leftrightarrow$ $-\frac{1}{2} < x+1 < \frac{1}{2}$ $\Leftrightarrow$ $-\frac{3}{2} < x < -\frac{1}{2}$.

In that case, the sum of the series is $\frac{a}{1-r} = \frac{1}{1-2(x+1)} = \frac{1}{-1-2x}$ or $\frac{-1}{2x+1}$.

38. $|CD| = b\sin\theta$, $|DE| = |CD|\sin\theta = b\sin^2\theta$, $|EF| = |DE|\sin\theta = b\sin^3\theta$, ... Therefore,

$|CD| + |DE| + |EF| + |FG| + \cdots = b\sum_{n=1}^{\infty} \sin^n\theta = b\left(\frac{\sin\theta}{1-\sin\theta}\right)$ since this is a geometric series with $r = \sin\theta$ and $|\sin\theta| < 1$ $\left[\text{because } 0 < \theta < \frac{\pi}{2}\right]$.