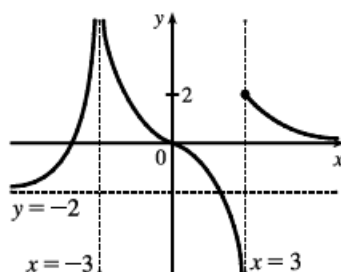


2. $\lim_{x \rightarrow -\infty} f(x) = -2$, $\lim_{x \rightarrow \infty} f(x) = 0$, $\lim_{x \rightarrow 3} f(x) = \infty$,

$\lim_{x \rightarrow 3^-} f(x) = -\infty$, $\lim_{x \rightarrow 3^+} f(x) = 2$,

f is continuous from the right at 3



8. $\lim_{t \rightarrow 2} \frac{t^2 - 4}{t^3 - 8} = \lim_{t \rightarrow 2} \frac{(t+2)(t-2)}{(t-2)(t^2 + 2t + 4)} = \lim_{t \rightarrow 2} \frac{t+2}{t^2 + 2t + 4} = \frac{2+2}{4+4+4} = \frac{4}{12} = \frac{1}{3}$

19. Let $t = 1/x$. Then as $x \rightarrow 0^+$, $t \rightarrow \infty$, and $\lim_{x \rightarrow 0^+} \tan^{-1}(1/x) = \lim_{t \rightarrow \infty} \tan^{-1} t = \frac{\pi}{2}$.

20. $\lim_{x \rightarrow 1} \left(\frac{1}{x-1} + \frac{1}{x^2 - 3x + 2} \right) = \lim_{x \rightarrow 1} \left[\frac{1}{x-1} + \frac{1}{(x-1)(x-2)} \right] = \lim_{x \rightarrow 1} \left[\frac{x-2}{(x-1)(x-2)} + \frac{1}{(x-1)(x-2)} \right]$
 $= \lim_{x \rightarrow 1} \left[\frac{x-1}{(x-1)(x-2)} \right] = \lim_{x \rightarrow 1} \frac{1}{x-2} = \frac{1}{1-2} = -1$

23. Since $2x - 1 \leq f(x) \leq x^2$ for $0 < x < 3$ and $\lim_{x \rightarrow 1} (2x - 1) = 1 = \lim_{x \rightarrow 1} x^2$, we have $\lim_{x \rightarrow 1} f(x) = 1$ by the Squeeze Theorem.

26. Given $\varepsilon > 0$ we must find $\delta > 0$ so that if $0 < |x - 0| < \delta$, then $|\sqrt[3]{x} - 0| < \varepsilon$. Now $|\sqrt[3]{x} - 0| = |\sqrt[3]{x}| < \varepsilon \Rightarrow$

$|x| = |\sqrt[3]{x}|^3 < \varepsilon^3$. So take $\delta = \varepsilon^3$. Then $0 < |x - 0| = |x| < \varepsilon^3 \Rightarrow |\sqrt[3]{x} - 0| = |\sqrt[3]{x}| = \sqrt[3]{|x|} < \sqrt[3]{\varepsilon^3} = \varepsilon$.

Therefore, by the definition of a limit, $\lim_{x \rightarrow 0} \sqrt[3]{x} = 0$.

30. (a) $g(x) = 2x - x^2$ if $0 \leq x \leq 2$, $g(x) = 2 - x$ if $2 < x \leq 3$, $g(x) = x - 4$ if $3 < x < 4$, $g(x) = \pi$ if $x \geq 4$.

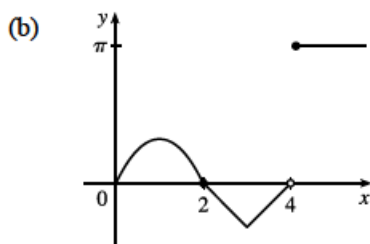
Therefore, $\lim_{x \rightarrow 2^-} g(x) = \lim_{x \rightarrow 2^-} (2x - x^2) = 0$ and $\lim_{x \rightarrow 2^+} g(x) = \lim_{x \rightarrow 2^+} (2 - x) = 0$. Thus, $\lim_{x \rightarrow 2} g(x) = 0 = g(2)$,

so g is continuous at 2. $\lim_{x \rightarrow 3^-} g(x) = \lim_{x \rightarrow 3^-} (2 - x) = -1$ and $\lim_{x \rightarrow 3^+} g(x) = \lim_{x \rightarrow 3^+} (x - 4) = -1$. Thus,

$\lim_{x \rightarrow 3} g(x) = -1 = g(3)$, so g is continuous at 3. $\lim_{x \rightarrow 4^-} g(x) = \lim_{x \rightarrow 4^-} (x - 4) = 0$ and $\lim_{x \rightarrow 4^+} g(x) = \lim_{x \rightarrow 4^+} \pi = \pi$.

Thus, $\lim_{x \rightarrow 4} g(x)$ does not exist, so g is discontinuous at 4. But $\lim_{x \rightarrow 4^+} g(x) = \pi = g(4)$, so g is continuous from the

right at 4.



32. $x^2 - 9$ is continuous on \mathbb{R} since it is a polynomial and \sqrt{x} is continuous on $[0, \infty)$ by Theorem 7 in Section 2.5, so the composition $\sqrt{x^2 - 9}$ is continuous on $\{x \mid x^2 - 9 \geq 0\} = (-\infty, -3] \cup [3, \infty)$ by Theorem 9. Note that $x^2 - 2 \neq 0$ on this set and so the quotient function $g(x) = \frac{\sqrt{x^2 - 9}}{x^2 - 2}$ is continuous on its domain, $(-\infty, -3] \cup [3, \infty)$ by Theorem 4.

34. $f(x) = \cos \sqrt{x} - e^x + 2$ is continuous on the interval $[0, 1]$, $f(0) = 2$, and $f(1) \approx -0.2$. Since $-0.2 < 0 < 2$, there is a number c in $(0, 1)$ such that $f(c) = 0$ by the Intermediate Value Theorem. Thus, there is a root of the equation $\cos \sqrt{x} - e^x + 2 = 0$, or $\cos \sqrt{x} = e^x - 2$, in the interval $(0, 1)$.

36. For a general point with x -coordinate a , we have

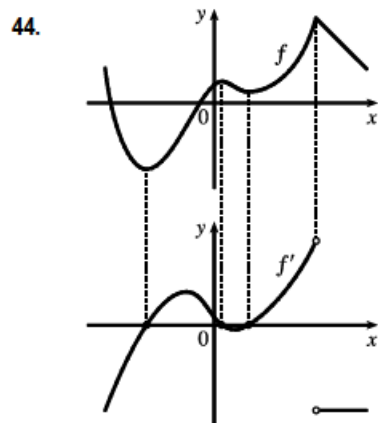
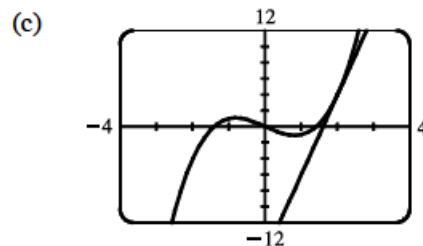
$$\begin{aligned} m &= \lim_{x \rightarrow a} \frac{2/(1-3x) - 2/(1-3a)}{x-a} = \lim_{x \rightarrow a} \frac{2(1-3a) - 2(1-3x)}{(1-3a)(1-3x)(x-a)} = \lim_{x \rightarrow a} \frac{6(x-a)}{(1-3a)(1-3x)(x-a)} \\ &= \lim_{x \rightarrow a} \frac{6}{(1-3a)(1-3x)} = \frac{6}{(1-3a)^2} \end{aligned}$$

For $a = 0$, $m = 6$ and $f(0) = 2$, so an equation of the tangent line is $y - 2 = 6(x - 0)$ or $y = 6x + 2$. For $a = -1$, $m = \frac{3}{8}$ and $f(-1) = \frac{1}{2}$, so an equation of the tangent line is $y - \frac{1}{2} = \frac{3}{8}(x + 1)$ or $y = \frac{3}{8}x + \frac{7}{8}$.

39. (a) $f'(2) = \lim_{x \rightarrow 2} \frac{f(x) - f(2)}{x - 2} = \lim_{x \rightarrow 2} \frac{x^3 - 2x - 4}{x - 2}$

$$= \lim_{x \rightarrow 2} \frac{(x-2)(x^2 + 2x + 2)}{x-2} = \lim_{x \rightarrow 2} (x^2 + 2x + 2) = 10$$

(b) $y - 4 = 10(x - 2)$ or $y = 10x - 16$



48. The graph of a has tangent lines with positive slope for $x < 0$ and negative slope for $x > 0$, and the values of c fit this pattern, so c must be the graph of the derivative of the function for a . The graph of c has horizontal tangent lines to the left and right of the x -axis and b has zeros at these points. Hence, b is the graph of the derivative of the function for c . Therefore, a is the graph of f , c is the graph of f' , and b is the graph of f'' .

50. (a) Drawing slope triangles, we obtain the following estimates: $F'(1950) \approx \frac{1.1}{10} = 0.11$, $F'(1965) \approx \frac{-1.6}{10} = -0.16$, and $F'(1987) \approx \frac{0.2}{10} = 0.02$.

(b) The rate of change of the average number of children born to each woman was increasing by 0.11 in 1950, decreasing by 0.16 in 1965, and increasing by 0.02 in 1987.

(c) There are many possible reasons:

- In the baby-boom era (post-WWII), there was optimism about the economy and family size was rising.
- In the baby-bust era, there was less economic optimism, and it was considered less socially responsible to have a large family.
- In the baby-boomlet era, there was increased economic optimism and a return to more conservative attitudes.

$$1. y = (x^2 + x^3)^4 \Rightarrow y' = 4(x^2 + x^3)^3(2x + 3x^2) = 4(x^2)^3(1 + x)^3x(2 + 3x) = 4x^7(x + 1)^3(3x + 2)$$

$$2. y = \frac{1}{\sqrt{x}} - \frac{1}{\sqrt[5]{x^3}} = x^{-1/2} - x^{-3/5} \Rightarrow y' = -\frac{1}{2}x^{-3/2} + \frac{3}{5}x^{-8/5} \text{ or } \frac{3}{5x\sqrt[5]{x^3}} - \frac{1}{2x\sqrt{x}} \text{ or } \frac{1}{10}x^{-8/5}(-5x^{1/10} + 6)$$

$$3. y = \frac{x^2 - x + 2}{\sqrt{x}} = x^{3/2} - x^{1/2} + 2x^{-1/2} \Rightarrow y' = \frac{3}{2}x^{1/2} - \frac{1}{2}x^{-1/2} - x^{-3/2} = \frac{3}{2}\sqrt{x} - \frac{1}{2\sqrt{x}} - \frac{1}{\sqrt{x^3}}$$