4. $y = \dfrac{\tan x}{1 + \cos x} \quad \Rightarrow \quad y' = \dfrac{(1 + \cos x)\sec^2 x - \tan x(-\sin x)}{(1 + \cos x)^2} = \dfrac{(1 + \cos x)\sec^2 x + \tan x\, \sin x}{(1 + \cos x)^2}$

8. $\dfrac{d}{dx}(xe^y) = \dfrac{d}{dx}(y \sin x) \quad \Rightarrow \quad xe^y y' + e^y \cdot 1 = y \cos x + \sin x \cdot y' \quad \Rightarrow \quad xe^y y' - \sin x \cdot y' = y \cos x - e^y \quad \Rightarrow$

$(xe^y - \sin x)y' = y \cos x - e^y \quad \Rightarrow \quad y' = \dfrac{y \cos x - e^y}{xe^y - \sin x}$

12. $y = (\arcsin 2x)^2 \quad \Rightarrow \quad y' = 2(\arcsin 2x) \cdot (\arcsin 2x)' = 2 \arcsin 2x \cdot \dfrac{1}{\sqrt{1 - (2x)^2}} \cdot 2 = \dfrac{4 \arcsin 2x}{\sqrt{1 - 4x^2}}$

16. $y = \left(\dfrac{u - 1}{u^2 + u + 1}\right)^4 \quad \Rightarrow$

$$y' = 4\left(\frac{u - 1}{u^2 + u + 1}\right)^3 \frac{d}{du}\left(\frac{u - 1}{u^2 + u + 1}\right) = 4\left(\frac{u - 1}{u^2 + u + 1}\right)^3 \frac{(u^2 + u + 1)(1) - (u - 1)(2u + 1)}{(u^2 + u + 1)^2}$$
$$= \frac{4(u - 1)^3}{(u^2 + u + 1)^3} \frac{u^2 + u + 1 - 2u^2 + u + 1}{(u^2 + u + 1)^2} = \frac{4(u - 1)^3(-u^2 + 2u + 2)}{(u^2 + u + 1)^5}$$

20. $y = e^{x \sec x} \quad \Rightarrow \quad y' = e^{x \sec x} \dfrac{d}{dx}(x \sec x) = e^{x \sec x}(x \sec x\, \tan x + \sec x \cdot 1) = \sec x\, e^{x \sec x}(x \tan x + 1)$

24. $y = \left(x + \sqrt{x}\right)^{-1/3} \quad \Rightarrow \quad y' = -\frac{1}{3}\left(x + \sqrt{x}\right)^{-4/3}\left(1 + \dfrac{1}{2\sqrt{x}}\right)$

28. $y = (\cos x)^x \quad \Rightarrow \quad \ln y = \ln(\cos x)^x = x \ln \cos x \quad \Rightarrow \quad \dfrac{y'}{y} = x \cdot \dfrac{1}{\cos x} \cdot (-\sin x) + \ln \cos x \cdot 1 \quad \Rightarrow$

$y' = (\cos x)^x(\ln \cos x - x \tan x)$

32. $y = e^{\cos x} + \cos(e^x) \quad \Rightarrow \quad y' = e^{\cos x}(-\sin x) + [-\sin(e^x) \cdot e^x] = -\sin x\, e^{\cos x} - e^x \sin(e^x)$

36. $y = \sqrt{t \ln(t^4)} \quad \Rightarrow$

$$y' = \frac{1}{2}[t \ln(t^4)]^{-1/2} \frac{d}{dt}[t \ln(t^4)] = \frac{1}{2\sqrt{t \ln(t^4)}} \cdot \left[1 \cdot \ln(t^4) + t \cdot \frac{1}{t^4} \cdot 4t^3\right] = \frac{1}{2\sqrt{t \ln(t^4)}} \cdot [\ln(t^4) + 4] = \frac{\ln(t^4) + 4}{2\sqrt{t \ln(t^4)}}$$

Or: Since y is only defined for $t > 0$, we can write $y = \sqrt{t \cdot 4 \ln t} = 2\sqrt{t \ln t}$. Then

$y' = 2 \cdot \dfrac{1}{2\sqrt{t \ln t}} \cdot \left(1 \cdot \ln t + t \cdot \dfrac{1}{t}\right) = \dfrac{\ln t + 1}{\sqrt{t \ln t}}$. This agrees with our first answer since

$\dfrac{\ln(t^4) + 4}{2\sqrt{t \ln(t^4)}} = \dfrac{4 \ln t + 4}{2\sqrt{t \cdot 4 \ln t}} = \dfrac{4(\ln t + 1)}{2 \cdot 2\sqrt{t \ln t}} = \dfrac{\ln t + 1}{\sqrt{t \ln t}}$.

40. $xe^y = y - 1 \quad \Rightarrow \quad xe^y y' + e^y = y' \quad \Rightarrow \quad e^y = y' - xe^y y' \quad \Rightarrow \quad y' = e^y/(1 - xe^y)$

44. $y = (\sin mx)/x \quad \Rightarrow \quad y' = (mx \cos mx - \sin mx)/x^2$

48. $y = x\tanh^{-1}\sqrt{x} \quad\Rightarrow\quad y' = \tanh^{-1}\sqrt{x} + x\,\dfrac{1}{1-\left(\sqrt{x}\right)^2}\,\dfrac{1}{2\sqrt{x}} = \tanh^{-1}\sqrt{x} + \dfrac{\sqrt{x}}{2(1-x)}$

52. $g(\theta) = \theta\sin\theta \quad\Rightarrow\quad g'(\theta) = \theta\cos\theta + \sin\theta\cdot 1 \quad\Rightarrow\quad g''(\theta) = \theta(-\sin\theta) + \cos\theta\cdot 1 + \cos\theta = 2\cos\theta - \theta\sin\theta,$

so $g''(\pi/6) = 2\cos(\pi/6) - (\pi/6)\sin(\pi/6) = 2\left(\sqrt{3}/2\right) - (\pi/6)(1/2) = \sqrt{3} - \pi/12.$

56. $\displaystyle\lim_{t\to 0}\frac{t^3}{\tan^3 2t} = \lim_{t\to 0}\frac{t^3\cos^3 2t}{\sin^3 2t} = \lim_{t\to 0}\cos^3 2t\cdot\frac{1}{8\dfrac{\sin^3 2t}{(2t)^3}} = \lim_{t\to 0}\frac{\cos^3 2t}{8\left(\lim\limits_{t\to 0}\dfrac{\sin 2t}{2t}\right)^3} = \frac{1}{8\cdot 1^3} = \frac{1}{8}$

60. $x^2 + 4xy + y^2 = 13 \quad\Rightarrow\quad 2x + 4(xy' + y\cdot 1) + 2yy' = 0 \quad\Rightarrow\quad x + 2xy' + 2y + yy' = 0 \quad\Rightarrow$

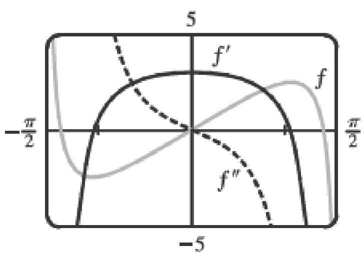
$2xy' + yy' = -x - 2y \quad\Rightarrow\quad y'(2x+y) = -x - 2y \quad\Rightarrow\quad y' = \dfrac{-x-2y}{2x+y}.$

At $(2,1)$, $y' = \dfrac{-2-2}{4+1} = -\dfrac{4}{5}$, so an equation of the tangent line is $y - 1 = -\frac{4}{5}(x-2)$, or $y = -\frac{4}{5}x + \frac{13}{5}$.

The slope of the normal line is $\frac{5}{4}$, so an equation of the normal line is $y - 1 = \frac{5}{4}(x-2)$, or $y = \frac{5}{4}x - \frac{3}{2}$.

64. (a) $f(x) = 4x - \tan x \quad\Rightarrow\quad f'(x) = 4 - \sec^2 x \quad\Rightarrow\quad f''(x) = -2\sec x\,(\sec x\,\tan x) = -2\sec^2 x\,\tan x.$

(b)



We can see that our answers are reasonable, since the graph of f' is 0 where f has a horizontal tangent, and the graph of f' is positive where f has tangents with positive slope and negative where f has tangents with negative slope. The same correspondence holds between the graphs of f' and f''.

68. (a) $\cos 2x = \cos^2 x - \sin^2 x \quad\Rightarrow\quad -2\sin 2x = -2\cos x\sin x - 2\sin x\cos x \quad\Leftrightarrow\quad \sin 2x = 2\sin x\cos x$

(b) $\sin(x+a) = \sin x\,\cos a + \cos x\,\sin a \quad\Rightarrow\quad \cos(x+a) = \cos x\,\cos a - \sin x\,\sin a.$

72. $f(x) = g(x^2) \quad\Rightarrow\quad f'(x) = g'(x^2)(2x) = 2xg'(x^2)$

76. $f(x) = e^{g(x)} \quad\Rightarrow\quad f'(x) = e^{g(x)}g'(x)$

80. $h(x) = \sqrt{\dfrac{f(x)}{g(x)}} \quad\Rightarrow\quad h'(x) = \dfrac{f'(x)\,g(x) - f(x)\,g'(x)}{2\sqrt{f(x)/g(x)}\,[g(x)]^2} = \dfrac{f'(x)\,g(x) - f(x)\,g'(x)}{2[g(x)]^{3/2}\sqrt{f(x)}}$

84. (a) The line $x - 4y = 1$ has slope $\frac{1}{4}$. A tangent to $y = e^x$ has slope $\frac{1}{4}$ when $y' = e^x = \frac{1}{4} \Rightarrow x = \ln \frac{1}{4} = -\ln 4$.

Since $y = e^x$, the y-coordinate is $\frac{1}{4}$ and the point of tangency is $\left(-\ln 4, \frac{1}{4}\right)$. Thus, an equation of the tangent line is $y - \frac{1}{4} = \frac{1}{4}(x + \ln 4)$ or $y = \frac{1}{4}x + \frac{1}{4}(\ln 4 + 1)$.

(b) The slope of the tangent at the point (a, e^a) is $\left.\dfrac{d}{dx} e^x \right|_{x=a} = e^a$. Thus, an equation of the tangent line is

$y - e^a = e^a(x - a)$. We substitute $x = 0$, $y = 0$ into this equation, since we want the line to pass through the origin:

$0 - e^a = e^a(0 - a) \Leftrightarrow -e^a = e^a(-a) \Leftrightarrow a = 1$. So an equation of the tangent line at the point $(a, e^a) = (1, e)$ is $y - e = e(x - 1)$ or $y = ex$.

88. (a) $x = \sqrt{b^2 + c^2 t^2} \Rightarrow v(t) = x' = \left[1/\left(2\sqrt{b^2 + c^2 t^2}\right)\right] 2c^2 t = c^2 t/\sqrt{b^2 + c^2 t^2} \Rightarrow$

$$a(t) = v'(t) = \frac{c^2\sqrt{b^2 + c^2 t^2} - c^2 t\left(c^2 t/\sqrt{b^2 + c^2 t^2}\right)}{b^2 + c^2 t^2} = \frac{b^2 c^2}{(b^2 + c^2 t^2)^{3/2}}$$

(b) $v(t) > 0$ for $t > 0$, so the particle always moves in the positive direction.

92. (a) $C(x) = 920 + 2x - 0.02x^2 + 0.00007x^3 \Rightarrow C'(x) = 2 - 0.04x + 0.00021x^2$

(b) $C'(100) = 2 - 4 + 2.1 = \$0.10/\text{unit}$. This value represents the rate at which costs are increasing as the hundredth unit is produced, and is the approximate cost of producing the 101st unit.

(c) The cost of producing the 101st item is $C(101) - C(100) = 990.10107 - 990 = \$0.10107$, slightly larger than $C'(100)$.

96. (a) If $y = u - 20$, $u(0) = 80 \Rightarrow y(0) = 80 - 20 = 60$, and the initial-value problem is $dy/dt = ky$ with $y(0) = 60$.

So the solution is $y(t) = 60e^{kt}$. Now $y(0.5) = 60e^{k(0.5)} = 60 - 20 \Rightarrow e^{0.5k} = \frac{40}{60} = \frac{2}{3} \Rightarrow k = 2\ln\frac{2}{3} = \ln\frac{4}{9}$,

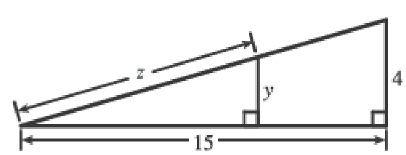
so $y(t) = 60e^{(\ln 4/9)t} = 60\left(\frac{4}{9}\right)^t$. Thus, $y(1) = 60\left(\frac{4}{9}\right)^1 = \frac{80}{3} = 26\frac{2}{3}$ °C and $u(1) = 46\frac{2}{3}$ °C.

(b) $u(t) = 40 \Rightarrow y(t) = 20$. $y(t) = 60\left(\dfrac{4}{9}\right)^t = 20 \Rightarrow \left(\dfrac{4}{9}\right)^t = \dfrac{1}{3} \Rightarrow t\ln\dfrac{4}{9} = \ln\dfrac{1}{3} \Rightarrow t = \dfrac{\ln\frac{1}{3}}{\ln\frac{4}{9}} \approx 1.35$ h

or 81.3 min.

100. We are given $dz/dt = 30$ ft/s. By similar triangles, $\dfrac{y}{z} = \dfrac{4}{\sqrt{241}} \Rightarrow$

$y = \dfrac{4}{\sqrt{241}}z$, so $\dfrac{dy}{dt} = \dfrac{4}{\sqrt{241}}\dfrac{dz}{dt} = \dfrac{120}{\sqrt{241}} \approx 7.7$ ft/s.



104. $y = x^3 - 2x^2 + 1 \Rightarrow dy = (3x^2 - 4x)\,dx$. When $x = 2$ and $dx = 0.2$, $dy = \left[3(2)^2 - 4(2)\right](0.2) = 0.8$.

108. $$\lim_{\theta \to \pi/3} \frac{\cos\theta - 0.5}{\theta - \pi/3} = \left[\frac{d}{d\theta}\cos\theta\right]_{\theta = \pi/3} = -\sin\frac{\pi}{3} = -\frac{\sqrt{3}}{2}$$

112. Let (b, c) be on the curve, that is, $b^{2/3} + c^{2/3} = a^{2/3}$. Now $x^{2/3} + y^{2/3} = a^{2/3} \Rightarrow \frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3}\frac{dy}{dx} = 0$, so $\frac{dy}{dx} = -\frac{y^{1/3}}{x^{1/3}} = -\left(\frac{y}{x}\right)^{1/3}$, so at (b, c) the slope of the tangent line is $-(c/b)^{1/3}$ and an equation of the tangent line is $y - c = -(c/b)^{1/3}(x - b)$ or $y = -(c/b)^{1/3}x + (c + b^{2/3}c^{1/3})$. Setting $y = 0$, we find that the x-intercept is $b^{1/3}c^{2/3} + b = b^{1/3}(c^{2/3} + b^{2/3}) = b^{1/3}a^{2/3}$ and setting $x = 0$ we find that the y-intercept is $c + b^{2/3}c^{1/3} = c^{1/3}(c^{2/3} + b^{2/3}) = c^{1/3}a^{2/3}$. So the length of the tangent line between these two points is

$$\begin{aligned}\sqrt{(b^{1/3}a^{2/3})^2 + (c^{1/3}a^{2/3})^2} &= \sqrt{b^{2/3}a^{4/3} + c^{2/3}a^{4/3}} = \sqrt{(b^{2/3} + c^{2/3})a^{4/3}} \\ &= \sqrt{a^{2/3}a^{4/3}} = \sqrt{a^2} = a = \text{constant}\end{aligned}$$