6. $\dfrac{d}{dx}\left(2\sqrt{x}+\sqrt{y}\right)=\dfrac{d}{dx}(3) \;\Rightarrow\; 2\cdot\dfrac{1}{2}x^{-1/2}+\dfrac{1}{2}y^{-1/2}\cdot y'=0 \;\Rightarrow\; \dfrac{1}{\sqrt{x}}+\dfrac{y'}{2\sqrt{y}}=0 \;\Rightarrow$

$\dfrac{y'}{2\sqrt{y}}=-\dfrac{1}{\sqrt{x}} \;\Rightarrow\; y'=-\dfrac{2\sqrt{y}}{\sqrt{x}}$

8. $\dfrac{d}{dx}\left(2x^3+x^2y-xy^3\right)=\dfrac{d}{dx}(2) \;\Rightarrow\; 6x^2+x^2\cdot y'+y\cdot 2x-(x\cdot 3y^2y'+y^3\cdot 1)=0 \;\Rightarrow$

$x^2\,y'-3xy^2\,y'=-6x^2-2xy+y^3 \;\Rightarrow\; (x^2-3xy^2)\,y'=-6x^2-2xy+y^3 \;\Rightarrow\; y'=\dfrac{-6x^2-2xy+y^3}{x^2-3xy^2}$

10. $\dfrac{d}{dx}(xe^y)=\dfrac{d}{dx}(x-y) \;\Rightarrow\; xe^y\,y'+e^y\cdot 1=1-y' \;\Rightarrow\; xe^y\,y'+y'=1-e^y \;\Rightarrow\; y'(xe^y+1)=1-e^y \;\Rightarrow$

$y'=\dfrac{1-e^y}{xe^y+1}$

12. $\dfrac{d}{dx}\cos(xy)=\dfrac{d}{dx}(1+\sin y) \;\Rightarrow\; -\sin(xy)(xy'+y\cdot 1)=\cos y\cdot y' \;\Rightarrow\; -xy'\sin(xy)-\cos y\cdot y'=y\sin(xy) \;\Rightarrow$

$y'[-x\sin(xy)-\cos y]=y\sin(xy) \;\Rightarrow\; y'=\dfrac{y\sin(xy)}{-x\sin(xy)-\cos y}=-\dfrac{y\sin(xy)}{x\sin(xy)+\cos y}$

18. $\dfrac{d}{dx}(x\sin y+y\sin x)=\dfrac{d}{dx}(1) \;\Rightarrow\; x\cos y\cdot y'+\sin y\cdot 1+y\cos x+\sin x\cdot y'=0 \;\Rightarrow$

$x\cos y\cdot y'+\sin x\cdot y'=-\sin y-y\cos x \;\Rightarrow\; y'(x\cos y+\sin x)=-\sin y-y\cos x \;\Rightarrow\; y'=\dfrac{-\sin y-y\cos x}{x\cos y+\sin x}$

22. $\dfrac{d}{dx}\left[g(x)+x\sin g(x)\right]=\dfrac{d}{dx}\left(x^2\right) \;\Rightarrow\; g'(x)+x\cos g(x)\cdot g'(x)+\sin g(x)\cdot 1=2x.$ If $x=0$, we have

$g'(0)+0+\sin g(0)=2(0) \;\Rightarrow\; g'(0)+\sin 0=0 \;\Rightarrow\; g'(0)+0=0 \;\Rightarrow\; g'(0)=0.$

30. $x^{2/3}+y^{2/3}=4 \;\Rightarrow\; \frac{2}{3}x^{-1/3}+\frac{2}{3}y^{-1/3}y'=0 \;\Rightarrow\; \dfrac{1}{\sqrt[3]{x}}+\dfrac{y'}{\sqrt[3]{y}}=0 \;\Rightarrow\; y'=-\dfrac{\sqrt[3]{y}}{\sqrt[3]{x}}.$ When $x=-3\sqrt{3}$

and $y=1$, we have $y'=-\dfrac{1}{\left(-3\sqrt{3}\right)^{1/3}}=-\dfrac{\left(-3\sqrt{3}\right)^{2/3}}{-3\sqrt{3}}=\dfrac{3}{3\sqrt{3}}=\dfrac{1}{\sqrt{3}}$, so an equation of the tangent line is

$y-1=\frac{1}{\sqrt{3}}\left(x+3\sqrt{3}\right)$ or $y=\frac{1}{\sqrt{3}}x+4.$

36. $\sqrt{x}+\sqrt{y}=1 \;\Rightarrow\; \dfrac{1}{2\sqrt{x}}+\dfrac{y'}{2\sqrt{y}}=0 \;\Rightarrow\; y'=-\dfrac{\sqrt{y}}{\sqrt{x}} \;\Rightarrow$

$$y''=-\frac{\sqrt{x}\left[\dfrac{1}{2\sqrt{y}}\right]y'-\sqrt{y}\left[\dfrac{1}{2\sqrt{x}}\right]}{x}=-\frac{\sqrt{x}\left(\dfrac{1}{\sqrt{y}}\right)\left(-\dfrac{\sqrt{y}}{\sqrt{x}}\right)-\sqrt{y}\left(\dfrac{1}{\sqrt{x}}\right)}{2x}=\frac{1+\dfrac{\sqrt{y}}{\sqrt{x}}}{2x}$$

$=\dfrac{\sqrt{x}+\sqrt{y}}{2x\sqrt{x}}=\dfrac{1}{2x\sqrt{x}}$ since x and y must satisfy the original equation, $\sqrt{x}+\sqrt{y}=1.$

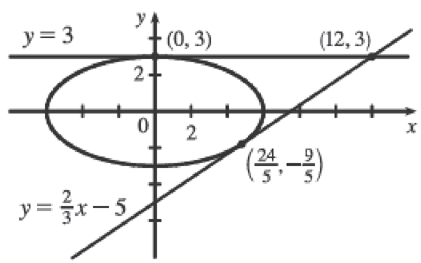
50. $y = \tan^{-1}(x^2) \quad \Rightarrow \quad y' = \dfrac{1}{1+(x^2)^2} \cdot \dfrac{d}{dx}(x^2) = \dfrac{1}{1+x^4} \cdot 2x = \dfrac{2x}{1+x^4}$

58. $y = \cos^{-1}(\sin^{-1} t) \quad \Rightarrow \quad y' = -\dfrac{1}{\sqrt{1-(\sin^{-1} t)^2}} \cdot \dfrac{d}{dt} \sin^{-1} t = -\dfrac{1}{\sqrt{1-(\sin^{-1} t)^2}} \cdot \dfrac{1}{\sqrt{1-t^2}}$

76. $x^2 + 4y^2 = 36 \quad \Rightarrow \quad 2x + 8yy' = 0 \quad \Rightarrow \quad y' = -\dfrac{x}{4y}$. Let (a, b) be a point on $x^2 + 4y^2 = 36$ whose tangent line passes through $(12, 3)$. The tangent line is then $y - 3 = -\dfrac{a}{4b}(x - 12)$, so $b - 3 = -\dfrac{a}{4b}(a - 12)$. Multiplying both sides by $4b$ gives $4b^2 - 12b = -a^2 + 12a$, so $4b^2 + a^2 = 12(a+b)$. But $4b^2 + a^2 = 36$, so $36 = 12(a+b) \quad \Rightarrow \quad a + b = 3 \quad \Rightarrow$ $b = 3 - a$. Substituting $3 - a$ for b into $a^2 + 4b^2 = 36$ gives $a^2 + 4(3-a)^2 = 36 \quad \Leftrightarrow \quad a^2 + 36 - 24a + 4a^2 = 36 \quad \Leftrightarrow$ $5a^2 - 24a = 0 \quad \Leftrightarrow \quad a(5a - 24) = 0$, so $a = 0$ or $a = \frac{24}{5}$. If $a = 0$, $b = 3 - 0 = 3$, and if $a = \frac{24}{5}$, $b = 3 - \frac{24}{5} = -\frac{9}{5}$.

So the two points on the ellipse are $(0, 3)$ and $\left(\frac{24}{5}, -\frac{9}{5}\right)$. Using $y - 3 = -\dfrac{a}{4b}(x - 12)$ with $(a, b) = (0, 3)$ gives us the tangent line $y - 3 = 0$ or $y = 3$. With $(a, b) = \left(\frac{24}{5}, -\frac{9}{5}\right)$, we have $y - 3 = -\frac{24/5}{4(-9/5)}(x - 12) \quad \Leftrightarrow \quad y - 3 = \frac{2}{3}(x - 12) \quad \Leftrightarrow \quad y = \frac{2}{3}x - 5$.

A graph of the ellipse and the tangent lines confirms our results.



2. $f(x) = x \ln x - x \quad \Rightarrow \quad f'(x) = x \cdot \dfrac{1}{x} + (\ln x) \cdot 1 - 1 = 1 + \ln x - 1 = \ln x$

8. $f(x) = \log_5(xe^x) \quad \Rightarrow \quad f'(x) = \dfrac{1}{xe^x \ln 5} \dfrac{d}{dx}(xe^x) = \dfrac{1}{xe^x \ln 5}(xe^x + e^x \cdot 1) = \dfrac{e^x(x+1)}{xe^x \ln 5} = \dfrac{x+1}{x \ln 5}$

Another solution: We can change the form of the function by first using logarithm properties.

$f(x) = \log_5(xe^x) = \log_5 x + \log_5 e^x \quad \Rightarrow \quad f'(x) = \dfrac{1}{x \ln 5} + \dfrac{1}{e^x \ln 5} \cdot e^x = \dfrac{1}{x \ln 5} + \dfrac{1}{\ln 5}$ or $\dfrac{1+x}{x \ln 5}$

10. $f(u) = \dfrac{u}{1 + \ln u} \quad \Rightarrow \quad f'(u) = \dfrac{(1+\ln u)(1) - u(1/u)}{(1+\ln u)^2} = \dfrac{1 + \ln u - 1}{(1+\ln u)^2} = \dfrac{\ln u}{(1+\ln u)^2}$

20. $H(z) = \ln \sqrt{\dfrac{a^2 - z^2}{a^2 + z^2}} = \ln\left(\dfrac{a^2 - z^2}{a^2 + z^2}\right)^{1/2} = \dfrac{1}{2} \ln\left(\dfrac{a^2 - z^2}{a^2 + z^2}\right) = \frac{1}{2}\ln(a^2 - z^2) - \frac{1}{2}\ln(a^2 + z^2) \quad \Rightarrow$

$$H'(z) = \frac{1}{2} \cdot \frac{1}{a^2 - z^2} \cdot (-2z) - \frac{1}{2} \cdot \frac{1}{a^2 + z^2} \cdot (2z) = \frac{z}{z^2 - a^2} - \frac{z}{z^2 + a^2} = \frac{z(z^2 + a^2) - z(z^2 - a^2)}{(z^2 - a^2)(z^2 + a^2)}$$

$$= \frac{z^3 + za^2 - z^3 + za^2}{(z^2 - a^2)(z^2 + a^2)} = \frac{2a^2 z}{z^4 - a^4}$$

26. $y = \ln(\sec x + \tan x) \quad \Rightarrow \quad y' = \dfrac{\sec x \, \tan x + \sec^2 x}{\sec x + \tan x} = \sec x \quad \Rightarrow \quad y'' = \sec x \, \tan x$

28. $f(x) = \sqrt{2 + \ln x} = (2 + \ln x)^{1/2} \quad \Rightarrow \quad f'(x) = \dfrac{1}{2}(2 + \ln x)^{-1/2} \cdot \dfrac{1}{x} = \dfrac{1}{2x\sqrt{2 + \ln x}}$

$\text{Dom}(f) = \{x \mid 2 + \ln x \geq 0\} = \{x \mid \ln x \geq -2\} = \{x \mid x \geq e^{-2}\} = [e^{-2}, \infty)$.

34. $y = x^2 \ln x \quad \Rightarrow \quad y' = x^2 \cdot \dfrac{1}{x} + (\ln x)(2x) \quad \Rightarrow \quad y'(1) = 1 + 0 = 1$, so an equation of a tangent line at $(1, 0)$ is $y - 0 = 1(x - 1)$, or $y = x - 1$.

42. $y = \sqrt{x}\, e^{x^2 - x}(x+1)^{2/3} \quad \Rightarrow \quad \ln y = \ln\left[x^{1/2} e^{x^2 - x}(x+1)^{2/3}\right] \quad \Rightarrow$

$\ln y = \frac{1}{2}\ln x + (x^2 - x) + \frac{2}{3}\ln(x+1) \quad \Rightarrow \quad \dfrac{1}{y}y' = \dfrac{1}{2} \cdot \dfrac{1}{x} + 2x - 1 + \dfrac{2}{3} \cdot \dfrac{1}{x+1} \quad \Rightarrow$

$y' = y\left(\dfrac{1}{2x} + 2x - 1 + \dfrac{2}{3x+3}\right) \quad \Rightarrow \quad y' = \sqrt{x}\, e^{x^2 - x}(x+1)^{2/3}\left(\dfrac{1}{2x} + 2x - 1 + \dfrac{2}{3x+3}\right)$

46. $y = \sqrt{x}^{\,x} \quad \Rightarrow \quad \ln y = \ln \sqrt{x}^{\,x} \quad \Rightarrow \quad \ln y = x \ln x^{1/2} \quad \Rightarrow \quad \ln y = \frac{1}{2}x \ln x \quad \Rightarrow \quad \dfrac{1}{y}y' = \dfrac{1}{2}x \cdot \dfrac{1}{x} + \ln x \cdot \dfrac{1}{2} \quad \Rightarrow$

$y' = y\left(\frac{1}{2} + \frac{1}{2}\ln x\right) \quad \Rightarrow \quad y' = \frac{1}{2}\sqrt{x}^{\,x}(1 + \ln x)$

50. $y = (\ln x)^{\cos x} \quad \Rightarrow \quad \ln y = \cos x \ln(\ln x) \quad \Rightarrow \quad \dfrac{y'}{y} = \cos x \cdot \dfrac{1}{\ln x} \cdot \dfrac{1}{x} + (\ln \ln x)(-\sin x) \quad \Rightarrow$

$y' = (\ln x)^{\cos x}\left(\dfrac{\cos x}{x \ln x} - \sin x \ln \ln x\right)$

52. $x^y = y^x \quad \Rightarrow \quad y \ln x = x \ln y \quad \Rightarrow \quad y \cdot \dfrac{1}{x} + (\ln x) \cdot y' = x \cdot \dfrac{1}{y} \cdot y' + \ln y \quad \Rightarrow \quad y' \ln x - \dfrac{x}{y}y' = \ln y - \dfrac{y}{x} \quad \Rightarrow$

$y' = \dfrac{\ln y - y/x}{\ln x - x/y}$

6. (a) The velocity v is positive when s is increasing, that is, on the intervals $(0, 1)$ and $(3, 4)$; and it is negative when s is decreasing, that is, on the interval $(1, 3)$. The acceleration a is positive when the graph of s is concave upward (v is increasing), that is, on the interval $(2, 4)$; and it is negative when the graph of s is concave downward (v is decreasing), that is, on the interval $(0, 2)$. The particle is speeding up on the interval $(1, 2)$ $[v < 0, a < 0]$ and on $(3, 4)$ $[v > 0, a > 0]$. The particle is slowing down on the interval $(0, 1)$ $[v > 0, a < 0]$ and on $(2, 3)$ $[v < 0, a > 0]$.

(b) The velocity v is positive on $(3, 4)$ and negative on $(0, 3)$. The acceleration a is positive on $(0, 1)$ and $(2, 4)$ and negative on $(1, 2)$. The particle is speeding up on the interval $(1, 2)$ $[v < 0, a < 0]$ and on $(3, 4)$ $[v > 0, a > 0]$. The particle is slowing down on the interval $(0, 1)$ $[v < 0, a > 0]$ and on $(2, 3)$ $[v < 0, a > 0]$.

8. (a) At maximum height the velocity of the ball is 0 ft/s. $v(t) = s'(t) = 80 - 32t = 0 \quad \Leftrightarrow \quad 32t = 80 \quad \Leftrightarrow \quad t = \frac{5}{2}$.

So the maximum height is $s\left(\frac{5}{2}\right) = 80\left(\frac{5}{2}\right) - 16\left(\frac{5}{2}\right)^2 = 200 - 100 = 100$ ft.

(b) $s(t) = 80t - 16t^2 = 96 \quad \Leftrightarrow \quad 16t^2 - 80t + 96 = 0 \quad \Leftrightarrow \quad 16(t^2 - 5t + 6) = 0 \quad \Leftrightarrow \quad 16(t-3)(t-2) = 0$.

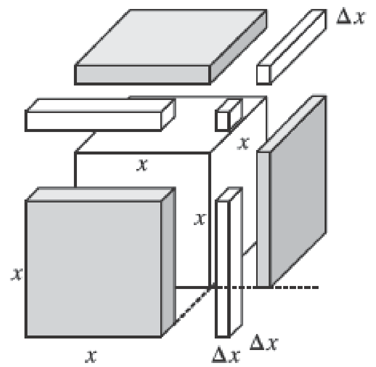
So the ball has a height of 96 ft on the way up at $t = 2$ and on the way down at $t = 3$. At these times the velocities are $v(2) = 80 - 32(2) = 16$ ft/s and $v(3) = 80 - 32(3) = -16$ ft/s, respectively.

12. (a) $V(x) = x^3 \quad \Rightarrow \quad \dfrac{dV}{dx} = 3x^2$. $\left.\dfrac{dV}{dx}\right|_{x=3} = 3(3)^2 = 27\ \text{mm}^3/\text{mm}$ is the rate at which the volume is increasing as x increases past 3 mm.

(b) The surface area is $S(x) = 6x^2$, so $V'(x) = 3x^2 = \frac{1}{2}(6x^2) = \frac{1}{2}S(x)$.

The figure suggests that if Δx is small, then the change in the volume of the cube is approximately half of its surface area (the area of 3 of the 6 faces) times Δx. From the figure, $\Delta V = 3x^2(\Delta x) + 3x(\Delta x)^2 + (\Delta x)^3$.

If Δx is small, then $\Delta V \approx 3x^2(\Delta x)$ and so $\Delta V/\Delta x \approx 3x^2$.



16. (a) Using $V(r) = \frac{4}{3}\pi r^3$, we find that the average rate of change is:

(i) $\dfrac{V(8) - V(5)}{8 - 5} = \dfrac{\frac{4}{3}\pi(512) - \frac{4}{3}\pi(125)}{3} = 172\pi\ \mu\text{m}^3/\mu\text{m}$

(ii) $\dfrac{V(6) - V(5)}{6 - 5} = \dfrac{\frac{4}{3}\pi(216) - \frac{4}{3}\pi(125)}{1} = 121.\overline{3}\pi\ \mu\text{m}^3/\mu\text{m}$

(iii) $\dfrac{V(5.1) - V(5)}{5.1 - 5} = \dfrac{\frac{4}{3}\pi(5.1)^3 - \frac{4}{3}\pi(5)^3}{0.1} = 102.01\overline{3}\pi\ \mu\text{m}^3/\mu\text{m}$

(b) $V'(r) = 4\pi r^2$, so $V'(5) = 100\pi\ \mu\text{m}^3/\mu\text{m}$.

(c) $V(r) = \frac{4}{3}\pi r^3 \quad \Rightarrow \quad V'(r) = 4\pi r^2 = S(r)$. By analogy with Exercise 13(c), we can say that the change in the volume of the spherical shell, ΔV, is approximately equal to its thickness, Δr, times the surface area of the inner sphere. Thus, $\Delta V \approx 4\pi r^2(\Delta r)$ and so $\Delta V/\Delta r \approx 4\pi r^2$.

22. (a) $D(t) = 7 + 5\cos[0.503(t - 6.75)] \quad \Rightarrow \quad D'(t) = -5\sin[0.503(t - 6.75)](0.503) = -2.515\sin[0.503(t - 6.75)]$.

At 3:00 AM, $t = 3$, and $D'(3) = -2.515\sin[0.503(-3.75)] \approx 2.39$ m/h (rising).

(b) At 6:00 AM, $t = 6$, and $D'(6) = -2.515\sin[0.503(-0.75)] \approx 0.93$ m/h (rising).

(c) At 9:00 AM, $t = 9$, and $D'(9) = -2.515\sin[0.503(2.25)] \approx -2.28$ m/h (falling).

(d) At noon, $t = 12$, and $D'(12) = -2.515\sin[0.503(5.25)] \approx -1.21$ m/h (falling).

26. $n = f(t) = \dfrac{a}{1 + be^{-0.7t}} \quad \Rightarrow \quad n' = -\dfrac{a \cdot be^{-0.7t}(-0.7)}{(1 + be^{-0.7t})^2}$ [Reciprocal Rule]. When $t = 0$, $n = 20$ and $n' = 12$.

$f(0) = 20 \quad \Rightarrow \quad 20 = \dfrac{a}{1+b} \quad \Rightarrow \quad a = 20(1+b)$. $f'(0) = 12 \quad \Rightarrow \quad 12 = \dfrac{0.7ab}{(1+b)^2} \quad \Rightarrow \quad 12 = \dfrac{0.7(20)(1+b)b}{(1+b)^2} \quad \Rightarrow$

$\dfrac{12}{14} = \dfrac{b}{1+b} \quad \Rightarrow \quad 6(1+b) = 7b \quad \Rightarrow \quad 6 + 6b = 7b \quad \Rightarrow \quad b = 6$ and $a = 20(1+6) = 140$. For the long run, we let t increase without bound. $\lim\limits_{t\to\infty} f(t) = \lim\limits_{t\to\infty} \dfrac{140}{1 + 6e^{-0.7t}} = \dfrac{140}{1 + 6 \cdot 0} = 140$, indicating that the yeast population stabilizes at 140 cells.

30. (a) (i) $f = \dfrac{1}{2L}\sqrt{\dfrac{T}{\rho}} = \left(\dfrac{1}{2}\sqrt{\dfrac{T}{\rho}}\right)L^{-1} \quad \Rightarrow \quad \dfrac{df}{dL} = -\left(\dfrac{1}{2}\sqrt{\dfrac{T}{\rho}}\right)L^{-2} = -\dfrac{1}{2L^2}\sqrt{\dfrac{T}{\rho}}$

(ii) $f = \dfrac{1}{2L}\sqrt{\dfrac{T}{\rho}} = \left(\dfrac{1}{2L\sqrt{\rho}}\right)T^{1/2} \quad \Rightarrow \quad \dfrac{df}{dT} = \dfrac{1}{2}\left(\dfrac{1}{2L\sqrt{\rho}}\right)T^{-1/2} = \dfrac{1}{4L\sqrt{T\rho}}$

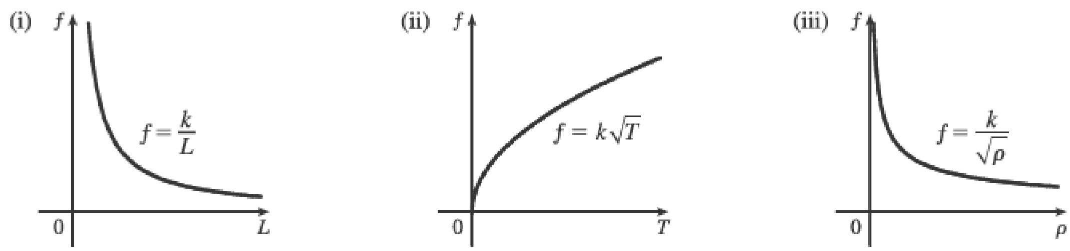
(iii) $f = \dfrac{1}{2L}\sqrt{\dfrac{T}{\rho}} = \left(\dfrac{\sqrt{T}}{2L}\right)\rho^{-1/2} \quad \Rightarrow \quad \dfrac{df}{d\rho} = -\dfrac{1}{2}\left(\dfrac{\sqrt{T}}{2L}\right)\rho^{-3/2} = -\dfrac{\sqrt{T}}{4L\rho^{3/2}}$

(b) *Note:* Illustrating tangent lines on the generic figures may help to explain the results.

(i) $\dfrac{df}{dL} < 0$ and L is decreasing $\quad \Rightarrow \quad f$ is increasing $\quad \Rightarrow \quad$ higher note

(ii) $\dfrac{df}{dT} > 0$ and T is increasing $\quad \Rightarrow \quad f$ is increasing $\quad \Rightarrow \quad$ higher note

(iii) $\dfrac{df}{d\rho} < 0$ and ρ is increasing $\quad \Rightarrow \quad f$ is decreasing $\quad \Rightarrow \quad$ lower note



36. (a) If $dP/dt = 0$, the population is stable (it is constant).

(b) $\dfrac{dP}{dt} = 0 \quad \Rightarrow \quad \beta P = r_0\left(1 - \dfrac{P}{P_c}\right)P \quad \Rightarrow \quad \dfrac{\beta}{r_0} = 1 - \dfrac{P}{P_c} \quad \Rightarrow \quad \dfrac{P}{P_c} = 1 - \dfrac{\beta}{r_0} \quad \Rightarrow \quad P = P_c\left(1 - \dfrac{\beta}{r_0}\right)$.

If $P_c = 10{,}000$, $r_0 = 5\% = 0.05$, and $\beta = 4\% = 0.04$, then $P = 10{,}000\left(1 - \frac{4}{5}\right) = 2000$.

(c) If $\beta = 0.05$, then $P = 10{,}000\left(1 - \frac{5}{5}\right) = 0$. There is no stable population.