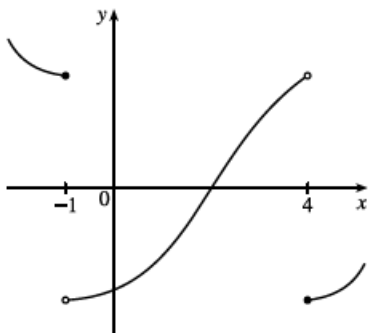


4. g is continuous on $[-4, -2)$, $(-2, 2)$, $[2, 4)$, $(4, 6)$, and $(6, 8)$.

6. The graph of $y = f(x)$ must have discontinuities at $x = -1$ and $x = 4$. It must show that

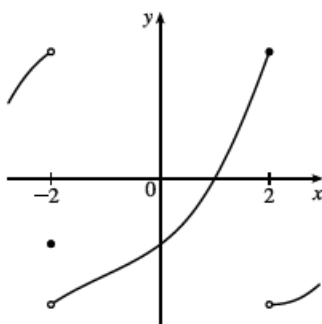
$$\lim_{x \rightarrow -1^-} f(x) = f(-1) \text{ and } \lim_{x \rightarrow 4^+} f(x) = f(4).$$



8. The graph of $y = f(x)$ must have a discontinuity at $x = -2$ with $\lim_{x \rightarrow -2^-} f(x) \neq f(-2)$ and

$$\lim_{x \rightarrow -2^+} f(x) \neq f(-2). \text{ It must also show that}$$

$$\lim_{x \rightarrow 2^-} f(x) = f(2) \text{ and } \lim_{x \rightarrow 2^+} f(x) \neq f(2).$$



10. (a) Continuous; at the location in question, the temperature changes smoothly as time passes, without any instantaneous jumps from one temperature to another.
- (b) Continuous; the temperature at a specific time changes smoothly as the distance due west from New York City increases, without any instantaneous jumps.
- (c) Discontinuous; as the distance due west from New York City increases, the altitude above sea level may jump from one height to another without going through all of the intermediate values— at a cliff, for example.
- (d) Discontinuous; as the distance traveled increases, the cost of the ride jumps in small increments.
- (e) Discontinuous; when the lights are switched on (or off), the current suddenly changes between 0 and some nonzero value, without passing through all of the intermediate values. This is debatable, though, depending on your definition of current.

$$\begin{aligned}
 12. \lim_{x \rightarrow 2} f(x) &= \lim_{x \rightarrow 2} (3x^4 - 5x + \sqrt[3]{x^2 + 4}) = 3 \lim_{x \rightarrow 2} x^4 - 5 \lim_{x \rightarrow 2} x + \sqrt[3]{\lim_{x \rightarrow 2} (x^2 + 4)} \\
 &= 3(2)^4 - 5(2) + \sqrt[3]{2^2 + 4} = 48 - 10 + 2 = 40 = f(2)
 \end{aligned}$$

By the definition of continuity, f is continuous at $a = 2$.

16. For $a < 3$, we have

$$\begin{aligned}
 \lim_{x \rightarrow a} g(x) &= \lim_{x \rightarrow a} 2\sqrt{3-x} \\
 &= 2 \lim_{x \rightarrow a} \sqrt{3-x} && \text{[Limit Law 3]} \\
 &= 2 \sqrt{\lim_{x \rightarrow a} (3-x)} && \text{[11]} \\
 &= 2 \sqrt{\lim_{x \rightarrow a} 3 - \lim_{x \rightarrow a} x} && \text{[2]} \\
 &= 2\sqrt{3-a} && \text{[7 and 8]} \\
 &= g(a)
 \end{aligned}$$

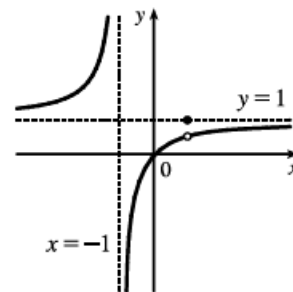
So g is continuous at $x = a$ for every a in $(-\infty, 3)$. Also, $\lim_{x \rightarrow 3^-} g(x) = 0 = g(3)$, so g is continuous from the left at 3.

Thus, g is continuous on $(-\infty, 3]$.

$$20. f(x) = \begin{cases} \frac{x^2 - x}{x^2 - 1} & \text{if } x \neq 1 \\ 1 & \text{if } x = 1 \end{cases}$$

$$\lim_{x \rightarrow 1} f(x) = \lim_{x \rightarrow 1} \frac{x^2 - x}{x^2 - 1} = \lim_{x \rightarrow 1} \frac{x(x-1)}{(x+1)(x-1)} = \lim_{x \rightarrow 1} \frac{x}{x+1} = \frac{1}{2},$$

but $f(1) = 1$, so f is discontinuous at 1.



$$24. f(x) = \frac{x^3 - 8}{x^2 - 4} = \frac{(x-2)(x^2 + 2x + 4)}{(x-2)(x+2)} = \frac{x^2 + 2x + 4}{x+2} \text{ for } x \neq 2. \text{ Since } \lim_{x \rightarrow 2} f(x) = \frac{4+4+4}{2+2} = 3, \text{ define } f(2) = 3.$$

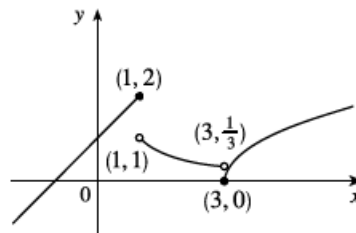
Then f is continuous at 2.

30. By Theorem 7, the trigonometric function $\tan x$ is continuous on its domain, $\{x \mid x \neq \frac{\pi}{2} + \pi n\}$. By Theorems 5(a), 7, and 9,

the composite function $\sqrt{4-x^2}$ is continuous on its domain $[-2, 2]$. By part 5 of Theorem 4, $B(x) = \frac{\tan x}{\sqrt{4-x^2}}$ is

continuous on its domain, $(-2, -\pi/2) \cup (-\pi/2, \pi/2) \cup (\pi/2, 2)$.

$$42. f(x) = \begin{cases} x + 1 & \text{if } x \leq 1 \\ 1/x & \text{if } 1 < x < 3 \\ \sqrt{x-3} & \text{if } x \geq 3 \end{cases}$$



f is continuous on $(-\infty, 1)$, $(1, 3)$, and $(3, \infty)$, where it is a polynomial, a rational function, and a composite of a root function with a polynomial,

respectively. Now $\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} (x + 1) = 2$ and $\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} (1/x) = 1$, so f is discontinuous at 1.

Since $f(1) = 2$, f is continuous from the left at 1. Also, $\lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^-} (1/x) = 1/3$, and

$\lim_{x \rightarrow 3^+} f(x) = \lim_{x \rightarrow 3^+} \sqrt{x-3} = 0 = f(3)$, so f is discontinuous at 3, but it is continuous from the right at 3.

$$46. f(x) = \begin{cases} \frac{x^2 - 4}{x - 2} & \text{if } x < 2 \\ ax^2 - bx + 3 & \text{if } 2 \leq x < 3 \\ 2x - a + b & \text{if } x \geq 3 \end{cases}$$

At $x = 2$: $\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^-} \frac{x^2 - 4}{x - 2} = \lim_{x \rightarrow 2^-} \frac{(x + 2)(x - 2)}{x - 2} = \lim_{x \rightarrow 2^-} (x + 2) = 2 + 2 = 4$

$$\lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2^+} (ax^2 - bx + 3) = 4a - 2b + 3$$

We must have $4a - 2b + 3 = 4$, or $4a - 2b = 1$ (1).

At $x = 3$: $\lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^-} (ax^2 - bx + 3) = 9a - 3b + 3$

$$\lim_{x \rightarrow 3^+} f(x) = \lim_{x \rightarrow 3^+} (2x - a + b) = 6 - a + b$$

We must have $9a - 3b + 3 = 6 - a + b$, or $10a - 4b = 3$ (2).

Now solve the system of equations by adding -2 times equation (1) to equation (2).

$$-8a + 4b = -2$$

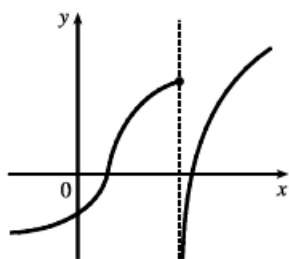
$$\frac{10a - 4b = 3}{2a = 1}$$

So $a = \frac{1}{2}$. Substituting $\frac{1}{2}$ for a in (1) gives us $-2b = -1$, so $b = \frac{1}{2}$ as well. Thus, for f to be continuous on $(-\infty, \infty)$,

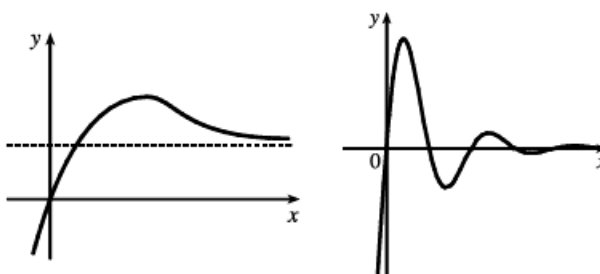
$$a = b = \frac{1}{2}.$$

52. $f(x) = \sqrt[3]{x} + x - 1$ is continuous on the interval $[0, 1]$, $f(0) = -1$, and $f(1) = 1$. Since $-1 < 0 < 1$, there is a number c in $(0, 1)$ such that $f(c) = 0$ by the Intermediate Value Theorem. Thus, there is a root of the equation $\sqrt[3]{x} + x - 1 = 0$, or $\sqrt[3]{x} = 1 - x$, in the interval $(0, 1)$.

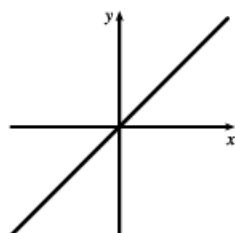
2. (a) The graph of a function can intersect a vertical asymptote in the sense that it can meet but not cross it.



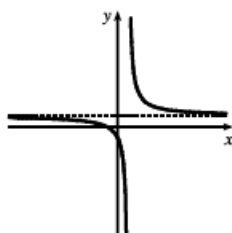
The graph of a function can intersect a horizontal asymptote. It can even intersect its horizontal asymptote an infinite number of times.



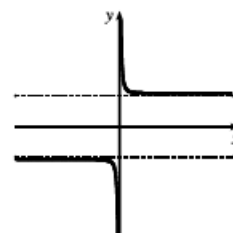
(b) The graph of a function can have 0, 1, or 2 horizontal asymptotes. Representative examples are shown.



No horizontal asymptote



One horizontal asymptote



Two horizontal asymptotes

4. (a) $\lim_{x \rightarrow \infty} g(x) = 2$

(b) $\lim_{x \rightarrow -\infty} g(x) = -1$

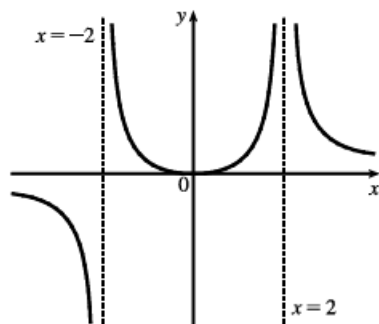
(c) $\lim_{x \rightarrow 0} g(x) = -\infty$

(d) $\lim_{x \rightarrow 2^-} g(x) = -\infty$

(e) $\lim_{x \rightarrow 2^+} g(x) = \infty$

(f) Vertical: $x = 0, x = 2$;
horizontal: $y = -1, y = 2$

6. $\lim_{x \rightarrow 2} f(x) = \infty, \lim_{x \rightarrow -2^+} f(x) = \infty,$
 $\lim_{x \rightarrow -2^-} f(x) = -\infty, \lim_{x \rightarrow -\infty} f(x) = 0,$
 $\lim_{x \rightarrow \infty} f(x) = 0, f(0) = 0$



$$16. \lim_{x \rightarrow \infty} \frac{1-x^2}{x^3-x+1} = \lim_{x \rightarrow \infty} \frac{(1-x^2)/x^3}{(x^3-x+1)/x^3} = \lim_{x \rightarrow \infty} \frac{1/x^3 - 1/x}{1 - 1/x^2 + 1/x^3}$$

$$= \frac{\lim_{x \rightarrow \infty} 1/x^3 - \lim_{x \rightarrow \infty} 1/x}{\lim_{x \rightarrow \infty} 1 - \lim_{x \rightarrow \infty} 1/x^2 + \lim_{x \rightarrow \infty} 1/x^3} = \frac{0-0}{1-0+0} = 0$$

$$\begin{aligned}
 24. \lim_{x \rightarrow -\infty} \frac{\sqrt{9x^6 - x}}{x^3 + 1} &= \lim_{x \rightarrow -\infty} \frac{\sqrt{9x^6 - x}/x^3}{(x^3 + 1)/x^3} = \frac{\lim_{x \rightarrow -\infty} -\sqrt{(9x^6 - x)/x^6}}{\lim_{x \rightarrow -\infty} (1 + 1/x^3)} \quad [\text{since } x^3 = -\sqrt{x^6} \text{ for } x < 0] \\
 &= \frac{\lim_{x \rightarrow -\infty} -\sqrt{9 - 1/x^5}}{\lim_{x \rightarrow -\infty} 1 + \lim_{x \rightarrow -\infty} (1/x^3)} = \frac{-\sqrt{\lim_{x \rightarrow -\infty} 9 - \lim_{x \rightarrow -\infty} (1/x^5)}}{1 + 0} = -\sqrt{9 - 0} = -3
 \end{aligned}$$

30. $\lim_{x \rightarrow \infty} (e^{-x} + 2 \cos 3x)$ does not exist. $\lim_{x \rightarrow \infty} e^{-x} = 0$, but $\lim_{x \rightarrow \infty} (2 \cos 3x)$ does not exist because the values of $2 \cos 3x$ oscillate between the values of -2 and 2 infinitely often, so the given limit does not exist.

34. Divide numerator and denominator by e^{3x} : $\lim_{x \rightarrow \infty} \frac{e^{3x} - e^{-3x}}{e^{3x} + e^{-3x}} = \lim_{x \rightarrow \infty} \frac{1 - e^{-6x}}{1 + e^{-6x}} = \frac{1 - 0}{1 + 0} = 1$

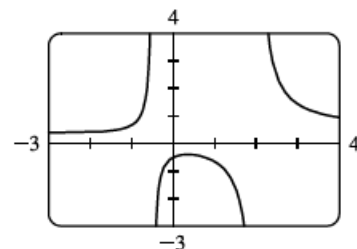
38. Let $t = \ln x$. As $x \rightarrow 0^+$, $t \rightarrow -\infty$. $\lim_{x \rightarrow 0^+} \tan^{-1}(\ln x) = \lim_{t \rightarrow -\infty} \tan^{-1} t = -\frac{\pi}{2}$ by (4).

$$\begin{aligned}
 42. \lim_{x \rightarrow \infty} \frac{x^2 + 1}{2x^2 - 3x - 2} &= \lim_{x \rightarrow \infty} \frac{\frac{x^2 + 1}{x^2}}{\frac{2x^2 - 3x - 2}{x^2}} = \lim_{x \rightarrow \infty} \frac{1 + \frac{1}{x^2}}{2 - \frac{3}{x} - \frac{2}{x^2}} = \frac{\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x^2}\right)}{\lim_{x \rightarrow \infty} \left(2 - \frac{3}{x} - \frac{2}{x^2}\right)} \\
 &= \frac{\lim_{x \rightarrow \infty} 1 + \lim_{x \rightarrow \infty} \frac{1}{x^2}}{\lim_{x \rightarrow \infty} 2 - \lim_{x \rightarrow \infty} \frac{3}{x} - \lim_{x \rightarrow \infty} \frac{2}{x^2}} = \frac{1 + 0}{2 - 0 - 0} = \frac{1}{2}, \quad \text{so } y = \frac{1}{2} \text{ is a horizontal asymptote.}
 \end{aligned}$$

$$y = f(x) = \frac{x^2 + 1}{2x^2 - 3x - 2} = \frac{x^2 + 1}{(2x + 1)(x - 2)}, \quad \text{so } \lim_{x \rightarrow (-1/2)^-} f(x) = \infty$$

because as $x \rightarrow (-1/2)^-$ the numerator is positive while the denominator approaches 0 through positive values. Similarly, $\lim_{x \rightarrow (-1/2)^+} f(x) = -\infty$,

$\lim_{x \rightarrow 2^-} f(x) = -\infty$, and $\lim_{x \rightarrow 2^+} f(x) = \infty$. Thus, $x = -\frac{1}{2}$ and $x = 2$ are vertical asymptotes. The graph confirms our work.



54. $y = f(x) = x^3(x + 2)^2(x - 1)$. The y -intercept is $f(0) = 0$. The x -intercepts are 0, -2 , and 1. There are sign changes at 0 and 1 (odd exponents on x and $x - 1$). There is no sign change at -2 . Also, $f(x) \rightarrow \infty$ as $x \rightarrow \infty$ because all three factors are large. And $f(x) \rightarrow \infty$ as $x \rightarrow -\infty$ because $x^3 \rightarrow -\infty$, $(x + 2)^2 \rightarrow \infty$, and $(x - 1) \rightarrow -\infty$. Note that the graph of f at $x = 0$ flattens out (looks like $y = -x^3$).

