2. $f(x) = \frac{1}{2}x^2 - 2x + 6 \quad\Rightarrow\quad F(x) = \frac{1}{2}\frac{x^3}{3} - 2\frac{x^2}{2} + 6x + C = \frac{1}{6}x^3 - x^2 + 6x + C$

8. $f(x) = x^{3.4} - 2x^{\sqrt{2}-1} \quad\Rightarrow\quad F(x) = \frac{x^{4.4}}{4.4} - 2\left(\frac{x^{\sqrt{2}}}{\sqrt{2}}\right) + C = \frac{5}{22}x^{4.4} - \sqrt{2}\,x^{\sqrt{2}} + C$

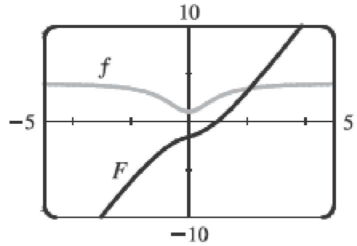
16. $r(\theta) = \sec\theta\,\tan\theta - 2e^\theta \quad\Rightarrow\quad R(\theta) = \sec\theta - 2e^\theta + C_n$ on the interval $\left(n\pi - \frac{\pi}{2}, n\pi + \frac{\pi}{2}\right)$.

24. $f(x) = 4 - 3\left(1+x^2\right)^{-1} = 4 - \frac{3}{1+x^2} \quad\Rightarrow\quad F(x) = 4x - 3\tan^{-1}x + C.$

$F(1) = 0 \quad\Rightarrow\quad 4 - 3\left(\frac{\pi}{4}\right) + C = 0 \quad\Rightarrow\quad C = \frac{3\pi}{4} - 4$, so

$F(x) = 4x - 3\tan^{-1}x + \frac{3\pi}{4} - 4$. Note that f is positive and F is increasing on $\mathbb{R}$.

Also, f has smaller values where the slopes of the tangent lines of F are smaller.



30. $f'''(t) = e^t + t^{-4}$ has domain $(-\infty, 0) \cup (0, \infty) \quad\Rightarrow\quad f''(t) = \begin{cases} e^t - \frac{1}{3}t^{-3} + C_1 & \text{if } t < 0 \\ e^t - \frac{1}{3}t^{-3} + C_2 & \text{if } t > 0 \end{cases} \quad\Rightarrow$

$f't) = \begin{cases} e^t + \frac{1}{6}t^{-2} + C_1 t + D_1 & \text{if } t < 0 \\ e^t + \frac{1}{6}t^{-2} + C_2 t + D_2 & \text{if } t > 0 \end{cases} \quad\Rightarrow\quad f(t) = \begin{cases} e^t - \frac{1}{6}t^{-1} + \frac{1}{2}C_1 t^2 + D_1 t + E_1 & \text{if } t < 0 \\ e^t - \frac{1}{6}t^{-1} + \frac{1}{2}C_2 t^2 + D_2 t + E_2 & \text{if } t > 0 \end{cases}$

36. $f'(x) = \frac{x^2-1}{x} = x - \frac{1}{x}$ has domain $(-\infty, 0) \cup (0, \infty) \quad\Rightarrow\quad f(x) = \begin{cases} \frac{1}{2}x^2 - \ln x + C_1 & \text{if } x > 0 \\ \frac{1}{2}x^2 - \ln(-x) + C_2 & \text{if } x < 0 \end{cases}$

$f(1) = \frac{1}{2} - \ln 1 + C_1 = \frac{1}{2} + C_1$ and $f(1) = \frac{1}{2} \quad\Rightarrow\quad C_1 = 0.$

$f(-1) = \frac{1}{2} - \ln 1 + C_2 = \frac{1}{2} + C_2$ and $f(-1) = 0 \quad\Rightarrow\quad C_2 = -\frac{1}{2}.$

Thus, $f(x) = \begin{cases} \frac{1}{2}x^2 - \ln x & \text{if } x > 0 \\ \frac{1}{2}x^2 - \ln(-x) - \frac{1}{2} & \text{if } x < 0 \end{cases}$

44. $f''(x) = x^3 + \sinh x \quad\Rightarrow\quad f'(x) = \frac{1}{4}x^4 + \cosh x + C \quad\Rightarrow\quad f(x) = \frac{1}{20}x^5 + \sinh x + Cx + D.\quad f(0) = D$ and $f(0) = 1 \quad\Rightarrow\quad D = 1$, so $f(x) = \frac{1}{20}x^5 + \sinh x + Cx + 1.\quad f(2) = \frac{32}{20} + \sinh 2 + 2C + 1$ and $f(2) = 2.6 \quad\Rightarrow$ $\sinh 2 + 2C = 0 \quad\Rightarrow\quad C = -\frac{1}{2}\sinh 2$, so $f(x) = \frac{1}{20}x^5 + \sinh x - \frac{1}{2}(\sinh 2)x + 1.$

50. $f'(x) = x^3 \quad\Rightarrow\quad f(x) = \frac{1}{4}x^4 + C.\quad x + y = 0 \quad\Rightarrow\quad y = -x \quad\Rightarrow\quad m = -1$. Now $m = f'(x) \quad\Rightarrow\quad -1 = x^3 \quad\Rightarrow$ $x = -1 \quad\Rightarrow\quad y = 1$ (from the equation of the tangent line), so $(-1, 1)$ is a point on the graph of f. From f, $1 = \frac{1}{4}(-1)^4 + C \quad\Rightarrow\quad C = \frac{3}{4}$. Therefore, the function is $f(x) = \frac{1}{4}x^4 + \frac{3}{4}$.

52. We know right away that c cannot be f's antiderivative, since the slope of c is not zero at the x-value where $f = 0$. Now f is positive when a is increasing and negative when a is decreasing, so a is the antiderivative of f.

60. $v(t) = s'(t) = 1.5\sqrt{t} \quad \Rightarrow \quad s(t) = t^{3/2} + C$. $s(4) = 8 + C$ and $s(4) = 10 \quad \Rightarrow \quad C = 2$, so $s(t) = t^{3/2} + 2$.

68. For the first ball, $s_1(t) = -16t^2 + 48t + 432$ from Example 7. For the second ball, $a(t) = -32 \quad \Rightarrow \quad v(t) = -32t + C$, but $v(1) = -32(1) + C = 24 \quad \Rightarrow \quad C = 56$, so $v(t) = -32t + 56 \quad \Rightarrow \quad s(t) = -16t^2 + 56t + D$, but $s(1) = -16(1)^2 + 56(1) + D = 432 \quad \Rightarrow \quad D = 392$, and $s_2(t) = -16t^2 + 56t + 392$. The balls pass each other when $s_1(t) = s_2(t) \quad \Rightarrow \quad -16t^2 + 48t + 432 = -16t^2 + 56t + 392 \quad \Leftrightarrow \quad 8t = 40 \quad \Leftrightarrow \quad t = 5$ s.

Another solution: From Exercise 66, we have $s_1(t) = -16t^2 + 48t + 432$ and $s_2(t) = -16t^2 + 24t + 432$.

We now want to solve $s_1(t) = s_2(t-1) \quad \Rightarrow \quad -16t^2 + 48t + 432 = -16(t-1)^2 + 24(t-1) + 432 \quad \Rightarrow$
$48t = 32t - 16 + 24t - 24 \quad \Rightarrow \quad 40 = 8t \quad \Rightarrow \quad t = 5$ s.

76. $a(t) = -16 \quad \Rightarrow \quad v(t) = -16t + v_0$ where v_0 is the car's speed (in ft/s) when the brakes were applied. The car stops when $-16t + v_0 = 0 \quad \Leftrightarrow \quad t = \frac{1}{16}v_0$. Now $s(t) = \frac{1}{2}(-16)t^2 + v_0 t = -8t^2 + v_0 t$. The car travels 200 ft in the time that it takes to stop, so $s\left(\frac{1}{16}v_0\right) = 200 \quad \Rightarrow \quad 200 = -8\left(\frac{1}{16}v_0\right)^2 + v_0\left(\frac{1}{16}v_0\right) = \frac{1}{32}v_0^2 \quad \Rightarrow \quad v_0^2 = 32 \cdot 200 = 6400 \quad \Rightarrow$
$v_0 = 80$ ft/s $[54.\overline{54}$ mi/h$]$.

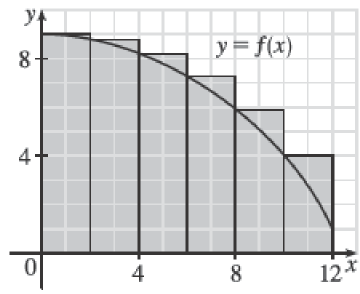
2. (a) (i) $L_6 = \sum_{i=1}^{6} f(x_{i-1})\Delta x \qquad [\Delta x = \frac{12-0}{6} = 2]$

$= 2[f(x_0) + f(x_1) + f(x_2) + f(x_3) + f(x_4) + f(x_5)]$

$= 2[f(0) + f(2) + f(4) + f(6) + f(8) + f(10)]$

$\approx 2(9 + 8.8 + 8.2 + 7.3 + 5.9 + 4.1)$

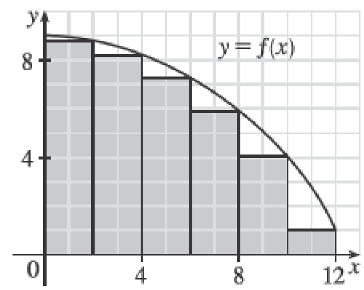
$= 2(43.3) = 86.6$



(ii) $R_6 = L_6 + 2 \cdot f(12) - 2 \cdot f(0)$

$\approx 86.6 + 2(1) - 2(9) = 70.6$

[Add area of rightmost lower rectangle
and subtract area of leftmost upper rectangle.]

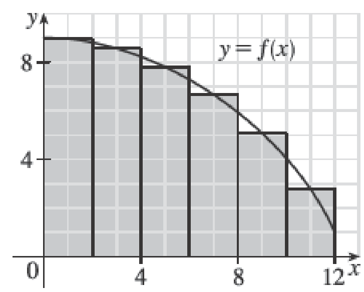


(iii) $M_6 = \sum_{i=1}^{6} f(x_i^*)\,\Delta x$

$= 2[f(1) + f(3) + f(5) + f(7) + f(9) + f(11)]$

$\approx 2(8.9 + 8.5 + 7.8 + 6.6 + 5.1 + 2.8)$

$= 2(39.7) = 79.4$



(b) Since f is *decreasing*, we obtain an *overestimate* by using *left* endpoints; that is, L_6.

(c) Since f is *decreasing*, we obtain an *underestimate* by using *right* endpoints; that is, R_6.

(d) M_6 gives the best estimate, since the area of each rectangle appears to be closer to the true area than the overestimates and underestimates in L_6 and R_6.

8. $f(x) = 1 + x^2$, $-1 \le x \le 1$, $\Delta x = 2/n$.

$\boldsymbol{n = 3}$: $\quad \textit{upper sum} = \left[f(-1) + f\left(\frac{1}{3}\right) + f(1)\right]\left(\frac{2}{3}\right)$

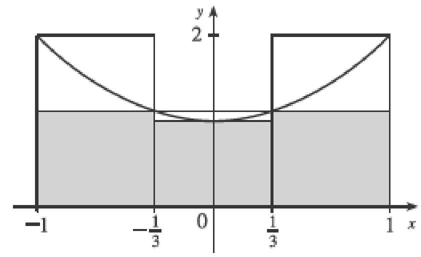
$= \left(2 + \frac{10}{9} + 2\right)\left(\frac{2}{3}\right)$

$= \frac{92}{27} \approx 3.41$

$\textit{lower sum} = \left[f\left(-\frac{1}{3}\right) + f(0) + f\left(\frac{1}{3}\right)\right]\left(\frac{2}{3}\right)$

$= \left(\frac{10}{9} + 1 + \frac{10}{9}\right)\left(\frac{2}{3}\right)$

$= \frac{58}{27} \approx 2.15$



$\boldsymbol{n = 4}$: $\quad \textit{upper sum} = \left[f(-1) + f\left(-\frac{1}{2}\right) + f\left(\frac{1}{2}\right) + f(1)\right]\left(\frac{2}{4}\right)$

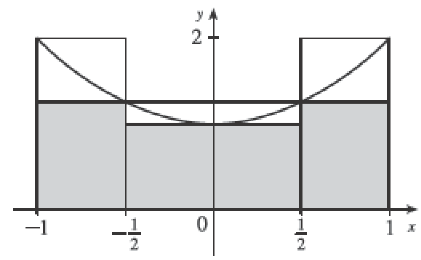
$= \left(2 + \frac{5}{4} + \frac{5}{4} + 2\right)\left(\frac{1}{2}\right)$

$= \frac{13}{4} = 3.25$

$\textit{lower sum} = \left[f\left(-\frac{1}{2}\right) + f(0) + f(0) + f\left(\frac{1}{2}\right)\right]\left(\frac{2}{4}\right)$

$= \left(\frac{5}{4} + 1 + 1 + \frac{5}{4}\right)\left(\frac{1}{2}\right)$

$= \frac{9}{4} = 2.25$



14. (a) $d \approx L_5 = (30 \text{ ft/s})(12 \text{ s}) + 28 \cdot 12 + 25 \cdot 12 + 22 \cdot 12 + 24 \cdot 12$

$= (30 + 28 + 25 + 22 + 24) \cdot 12 = 129 \cdot 12 = 1548 \text{ ft}$

(b) $d \approx R_5 = (28 + 25 + 22 + 24 + 27) \cdot 12 = 126 \cdot 12 = 1512 \text{ ft}$

(c) The estimates are neither lower nor upper estimates since v is neither an increasing nor a decreasing function of t.

18. For an increasing function, using left endpoints gives us an underestimate and using right endpoints results in an overestimate. We will use M_6 to get an estimate. $\Delta t = \frac{30-0}{6} = 5 \text{ s} = \frac{5}{3600} \text{ h} = \frac{1}{720} \text{ h}$.

$$M_6 = \tfrac{1}{720}[v(2.5) + v(7.5) + v(12.5) + v(17.5) + v(22.5) + v(27.5)]$$

$$= \tfrac{1}{720}(31.25 + 66 + 88 + 103.5 + 113.75 + 119.25) = \tfrac{1}{720}(521.75) \approx 0.725 \text{ km}$$

For a very rough check on the above calculation, we can draw a line from $(0, 0)$ to $(30, 120)$ and calculate the area of the triangle: $\frac{1}{2}(30)(120) = 1800$. Divide by 3600 to get 0.5, which is clearly an underestimate, making our midpoint estimate of 0.725 seem reasonable. Of course, answers will vary due to different readings of the graph.

20. $f(x) = x^2 + \sqrt{1 + 2x}$, $4 \le x \le 7$. $\Delta x = (7 - 4)/n = 3/n$ and $x_i = 4 + i\,\Delta x = 4 + 3i/n$.

$$A = \lim_{n\to\infty} R_n = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x = \lim_{n\to\infty} \sum_{i=1}^{n} \left[(4 + 3i/n)^2 + \sqrt{1 + 2(4 + 3i/n)}\right] \cdot \frac{3}{n}.$$

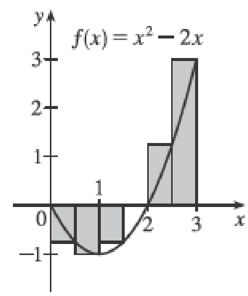
24. (a) $\Delta x = \dfrac{1-0}{n} = \dfrac{1}{n}$ and $x_i = 0 + i\,\Delta x = \dfrac{i}{n}$. $\quad A = \lim\limits_{n\to\infty} R_n = \lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} f(x_i)\,\Delta x = \lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \left(\dfrac{i}{n}\right)^3 \cdot \dfrac{1}{n}$.

(b) $\lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \dfrac{i^3}{n^3} \cdot \dfrac{1}{n} = \lim\limits_{n\to\infty} \dfrac{1}{n^4} \sum\limits_{i=1}^{n} i^3 = \lim\limits_{n\to\infty} \dfrac{1}{n^4} \left[\dfrac{n(n+1)}{2}\right]^2 = \lim\limits_{n\to\infty} \dfrac{(n+1)^2}{4n^2} = \dfrac{1}{4} \lim\limits_{n\to\infty} \left(1 + \dfrac{1}{n}\right)^2 = \dfrac{1}{4}$

2. $f(x) = x^2 - 2x,\ 0 \le x \le 3.\quad \Delta x = \dfrac{b-a}{n} = \dfrac{3-0}{6} = \dfrac{1}{2}.$

Since we are using right endpoints, $x_i^* = x_i$.

$$R_6 = \sum_{i=1}^{6} f(x_i)\,\Delta x$$
$$= (\Delta x)\,[f(x_1) + f(x_2) + f(x_3) + f(x_4) + f(x_5) + f(x_6)]$$
$$= \tfrac{1}{2}\left[f\left(\tfrac{1}{2}\right) + f(1) + f\left(\tfrac{3}{2}\right) + f(2) + f\left(\tfrac{5}{2}\right) + f(3)\right]$$
$$= \tfrac{1}{2}\left(-\tfrac{3}{4} - 1 - \tfrac{3}{4} + 0 + \tfrac{5}{4} + 3\right) = \tfrac{1}{2}\left(\tfrac{7}{4}\right) = \tfrac{7}{8}$$



The Riemann sum represents the sum of the areas of the two rectangles above the x-axis minus the sum of the areas of the three rectangles below the x-axis; that is, the *net area* of the rectangles with respect to the x-axis.

6. (a) $\displaystyle\int_{-2}^{4} g(x)\,dx \approx R_6 = [g(-1) + g(0) + g(1) + g(2) + g(3) + g(4)]\,\Delta x$

$$= \left[-\tfrac{3}{2} + 0 + \tfrac{3}{2} + \tfrac{1}{2} + (-1) + \tfrac{1}{2}\right](1) = 0$$

(b) $\displaystyle\int_{-2}^{4} g(x)\,dx \approx L_6 = [g(-2) + g(-1) + g(0) + g(1) + g(2) + g(3)]\,\Delta x$

$$= \left[0 + \left(-\tfrac{3}{2}\right) + 0 + \tfrac{3}{2} + \tfrac{1}{2} + (-1)\right](1) = -\tfrac{1}{2}$$

(c) $\displaystyle\int_{-2}^{4} g(x)\,dx \approx M_6 = \left[g\left(-\tfrac{3}{2}\right) + g\left(-\tfrac{1}{2}\right) + g\left(\tfrac{1}{2}\right) + g\left(\tfrac{3}{2}\right) + g\left(\tfrac{5}{2}\right) + g\left(\tfrac{7}{2}\right)\right]\Delta x$

$$= \left[-1 + (-1) + 1 + 1 + 0 + \left(-\tfrac{1}{2}\right)\right](1) = -\tfrac{1}{2}$$

12. $\Delta x = (5-1)/4 = 1$, so the endpoints are 1, 2, 3, 4, and 5, and the midpoints are 1.5, 2.5, 3.5, and 4.5. The Midpoint Rule gives

$$\int_1^5 x^2 e^{-x}\,dx \approx \sum_{i=1}^{4} f(\overline{x}_i)\,\Delta x = 1\left[(1.5)^2 e^{-1.5} + (2.5)^2 e^{-2.5} + (3.5)^2 e^{-3.5} + (4.5)^2 e^{-4.5}\right] \approx 1.6099.$$

18. On $[\pi, 2\pi]$, $\displaystyle\lim_{n\to\infty} \sum_{i=1}^{n} \frac{\cos x_i}{x_i}\,\Delta x = \int_{\pi}^{2\pi} \frac{\cos x}{x}\,dx.$

22. Note that $\Delta x = \dfrac{4-1}{n} = \dfrac{3}{n}$ and $x_i = 1 + i\,\Delta x = 1 + \dfrac{3i}{n}$.

$$\int_1^4 (x^2 - 4x + 2)\,dx = \lim_{n\to\infty} \sum_{i=1}^n f(x_i)\,\Delta x = \lim_{n\to\infty} \sum_{i=1}^n f\left(1 + \frac{3i}{n}\right)\frac{3}{n} = \lim_{n\to\infty} \frac{3}{n} \sum_{i=1}^n \left[\left(1+\frac{3i}{n}\right)^2 - 4\left(1+\frac{3i}{n}\right) + 2\right]$$

$$= \lim_{n\to\infty} \frac{3}{n} \sum_{i=1}^n \left[1 + \frac{6i}{n} + \frac{9i^2}{n^2} - 4 - \frac{12i}{n} + 2\right] = \lim_{n\to\infty} \frac{3}{n} \sum_{i=1}^n \left[\frac{9i^2}{n^2} - \frac{6i}{n} - 1\right]$$

$$= \lim_{n\to\infty} \frac{3}{n}\left[\frac{9}{n^2}\sum_{i=1}^n i^2 - \frac{6}{n}\sum_{i=1}^n i - \sum_{i=1}^n 1\right]$$

$$= \lim_{n\to\infty} \left[\frac{27}{n^3}\frac{n(n+1)(2n+1)}{6} - \frac{18}{n^2}\frac{n(n+1)}{2} - \frac{3}{n}\cdot n(1)\right]$$

$$= \lim_{n\to\infty} \left[\frac{9}{2}\frac{(n+1)(2n+1)}{n^2} - 9\frac{n+1}{n} - 3\right] = \lim_{n\to\infty} \left[\frac{9}{2}\frac{n+1}{n}\frac{2n+1}{n} - 9\left(1+\frac{1}{n}\right) - 3\right]$$

$$= \lim_{n\to\infty} \left[\frac{9}{2}\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right) - 9\left(1+\frac{1}{n}\right) - 3\right] = \frac{9}{2}(1)(2) - 9(1) - 3 = -3$$

30. $\Delta x = \dfrac{10-1}{n} = \dfrac{9}{n}$ and $x_i = 1 + i\,\Delta x = 1 + \dfrac{9i}{n}$, so

$$\int_1^{10} (x - 4\ln x)\,dx = \lim_{n\to\infty} R_n = \lim_{n\to\infty} \sum_{i=1}^n \left[\left(1+\frac{9i}{n}\right) - 4\ln\left(1+\frac{9i}{n}\right)\right]\cdot\frac{9}{n}.$$

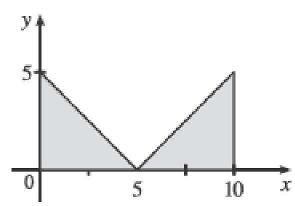
34. (a) $\int_0^2 g(x)\,dx = \frac{1}{2}\cdot 4\cdot 2 = 4$ [area of a triangle]

(b) $\int_2^6 g(x)\,dx = -\frac{1}{2}\pi(2)^2 = -2\pi$ [negative of the area of a semicircle]

(c) $\int_6^7 g(x)\,dx = \frac{1}{2}\cdot 1\cdot 1 = \frac{1}{2}$ [area of a triangle]

$\int_0^7 g(x)\,dx = \int_0^2 g(x)\,dx + \int_2^6 g(x)\,dx + \int_6^7 g(x)\,dx = 4 - 2\pi + \frac{1}{2} = 4.5 - 2\pi$

40. $\int_0^{10} |x-5|\,dx$ can be interpreted as the sum of the areas of the two shaded triangles; that is, $2\left(\frac{1}{2}\right)(5)(5) = 25$.



50. If $f(x) = \begin{cases} 3 & \text{for } x < 3 \\ x & \text{for } x \geq 3 \end{cases}$, then $\int_0^5 f(x)\,dx$ can be interpreted as the area of the shaded region, which consists of a 5-by-3 rectangle surmounted by an isosceles right triangle whose legs have length 2. Thus, $\int_0^5 f(x)\,dx = 5(3) + \frac{1}{2}(2)(2) = 17$.