2. (a) Slope $= \frac{2948-2530}{42-36} = \frac{418}{6} \approx 69.67$

(b) Slope $= \frac{2948-2661}{42-38} = \frac{287}{4} = 71.75$

(c) Slope $= \frac{2948-2806}{42-40} = \frac{142}{2} = 71$

(d) Slope $= \frac{3080-2948}{44-42} = \frac{132}{2} = 66$

From the data, we see that the patient's heart rate is decreasing from 71 to 66 heartbeats/minute after 42 minutes. After being stable for a while, the patient's heart rate is dropping.

6. (a) $y = y(t) = 10t - 1.86t^2$. At $t = 1$, $y = 10(1) - 1.86(1)^2 = 8.14$. The average velocity between times 1 and $1+h$ is

$$v_{\text{ave}} = \frac{y(1+h) - y(1)}{(1+h) - 1} = \frac{\left[10(1+h) - 1.86(1+h)^2\right] - 8.14}{h} = \frac{6.28h - 1.86h^2}{h} = 6.28 - 1.86h, \text{ if } h \neq 0.$$

(i) $[1, 2]$: $h = 1$, $v_{\text{ave}} = 4.42$ m/s

(ii) $[1, 1.5]$: $h = 0.5$, $v_{\text{ave}} = 5.35$ m/s

(iii) $[1, 1.1]$: $h = 0.1$, $v_{\text{ave}} = 6.094$ m/s

(iv) $[1, 1.01]$: $h = 0.01$, $v_{\text{ave}} = 6.2614$ m/s

(v) $[1, 1.001]$: $h = 0.001$, $v_{\text{ave}} = 6.27814$ m/s

(b) The instantaneous velocity when $t = 1$ (h approaches 0) is 6.28 m/s.

8. (a) (i) $s = s(t) = 2\sin \pi t + 3\cos \pi t$. On the interval $[1, 2]$, $v_{\text{ave}} = \dfrac{s(2) - s(1)}{2 - 1} = \dfrac{3 - (-3)}{1} = 6$ cm/s.

(ii) On the interval $[1, 1.1]$, $v_{\text{ave}} = \dfrac{s(1.1) - s(1)}{1.1 - 1} \approx \dfrac{-3.471 - (-3)}{0.1} = -4.71$ cm/s.

(iii) On the interval $[1, 1.01]$, $v_{\text{ave}} = \dfrac{s(1.01) - s(1)}{1.01 - 1} \approx \dfrac{-3.0613 - (-3)}{0.01} = -6.13$ cm/s.

(iv) On the interval $[1, 1.001]$, $v_{\text{ave}} = \dfrac{s(1.001) - s(1)}{1.001 - 1} \approx \dfrac{-3.00627 - (-3)}{1.001 - 1} = -6.27$ cm/s.

(b) The instantaneous velocity of the particle when $t = 1$ appears to be about -6.3 cm/s.

4. (a) As x approaches 2 from the left, the values of $f(x)$ approach 3, so $\lim\limits_{x \to 2^-} f(x) = 3$.

(b) As x approaches 2 from the right, the values of $f(x)$ approach 1, so $\lim\limits_{x \to 2^+} f(x) = 1$.

(c) $\lim\limits_{x \to 2} f(x)$ does not exist since the left-hand limit does not equal the right-hand limit.

(d) When $x = 2$, $y = 3$, so $f(2) = 3$.

(e) As x approaches 4, the values of $f(x)$ approach 4, so $\lim\limits_{x \to 4} f(x) = 4$.

(f) There is no value of $f(x)$ when $x = 4$, so $f(4)$ does not exist.

8. (a) $\lim\limits_{x \to 2} R(x) = -\infty$

(b) $\lim\limits_{x \to 5} R(x) = \infty$

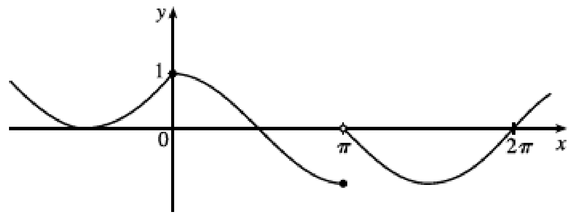
(c) $\lim\limits_{x \to -3^-} R(x) = -\infty$

(d) $\lim\limits_{x \to -3^+} R(x) = \infty$

(e) The equations of the vertical asymptotes are $x = -3$, $x = 2$, and $x = 5$.

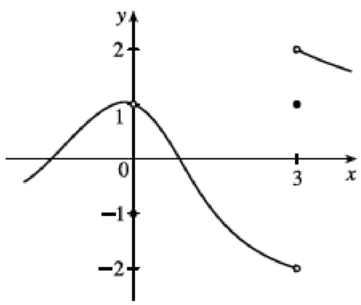
12. From the graph of

$$f(x) = \begin{cases} 1 + \sin x & \text{if } x < 0 \\ \cos x & \text{if } 0 \le x \le \pi, \\ \sin x & \text{if } x > \pi \end{cases}$$



we see that $\lim_{x \to a} f(x)$ exists for all a except $a = \pi$. Notice that the right and left limits are different at $a = \pi$.

16. $\lim_{x \to 0} f(x) = 1$, $\lim_{x \to 3^-} f(x) = -2$, $\lim_{x \to 3^+} f(x) = 2$, $f(0) = -1$, $f(3) = 1$



20. For $f(x) = \dfrac{x^2 - 2x}{x^2 - x - 2}$:

x	$f(x)$
0	0
−0.5	−1
−0.9	−9
−0.95	−19
−0.99	−99
−0.999	−999

x	$f(x)$
−2	2
−1.5	3
−1.1	11
−1.01	101
−1.001	1001

It appears that $\lim_{x \to -1} \dfrac{x^2 - 2x}{x^2 - x - 2}$ *does not exist* since $f(x) \to \infty$ as $x \to -1^-$ and $f(x) \to -\infty$ as $x \to -1^+$.

22. For $f(h) = \dfrac{(2+h)^5 - 32}{h}$:

h	$f(h)$
0.5	131.312500
0.1	88.410100
0.01	80.804010
0.001	80.080040
0.0001	80.008000

h	$f(h)$
-0.5	48.812500
-0.1	72.390100
-0.01	79.203990
-0.001	79.920040
-0.0001	79.992000

It appears that $\lim\limits_{h \to 0} \dfrac{(2+h)^5 - 32}{h} = 80$.

36. $\lim\limits_{x \to 2^-} \dfrac{x^2 - 2x}{x^2 - 4x + 4} = \lim\limits_{x \to 2^-} \dfrac{x(x-2)}{(x-2)^2} = \lim\limits_{x \to 2^-} \dfrac{x}{x-2} = -\infty$ since the numerator is positive and the denominator approaches 0 through negative values as $x \to 2^-$.

2. (a) $\lim\limits_{x \to 2} [f(x) + g(x)] = \lim\limits_{x \to 2} f(x) + \lim\limits_{x \to 2} g(x) = 2 + 0 = 2$

(b) $\lim\limits_{x \to 1} g(x)$ does not exist since its left- and right-hand limits are not equal, so the given limit does not exist.

(c) $\lim\limits_{x \to 0} [f(x) g(x)] = \lim\limits_{x \to 0} f(x) \cdot \lim\limits_{x \to 0} g(x) = 0 \cdot 1.3 = 0$

(d) Since $\lim\limits_{x \to -1} g(x) = 0$ and g is in the denominator, but $\lim\limits_{x \to -1} f(x) = -1 \neq 0$, the given limit does not exist.

(e) $\lim\limits_{x \to 2} x^3 f(x) = \left[\lim\limits_{x \to 2} x^3\right]\left[\lim\limits_{x \to 2} f(x)\right] = 2^3 \cdot 2 = 16$

(f) $\lim\limits_{x \to 1} \sqrt{3 + f(x)} = \sqrt{3 + \lim\limits_{x \to 1} f(x)} = \sqrt{3 + 1} = 2$

6. $$\begin{aligned}\lim_{u \to -2} \sqrt{u^4 + 3u + 6} &= \sqrt{\lim_{u \to -2} (u^4 + 3u + 6)} && [11] \\ &= \sqrt{\lim_{u \to -2} u^4 + 3 \lim_{u \to -2} u + \lim_{u \to -2} 6} && [1, 2, \text{and } 3] \\ &= \sqrt{(-2)^4 + 3(-2) + 6} && [9, 8, \text{and } 7] \\ &= \sqrt{16 - 6 + 6} = \sqrt{16} = 4\end{aligned}$$

8. $\lim_{t\to 2}\left(\frac{t^2-2}{t^3-3t+5}\right)^2 = \left(\lim_{t\to 2}\frac{t^2-2}{t^3-3t+5}\right)^2$ [Limit Law 6]

$= \left(\frac{\lim_{t\to 2}(t^2-2)}{\lim_{t\to 2}(t^3-3t+5)}\right)^2$ [5]

$= \left(\frac{\lim_{t\to 2}t^2-\lim_{t\to 2}2}{\lim_{t\to 2}t^3-3\lim_{t\to 2}t+\lim_{t\to 2}5}\right)^2$ [1, 2, and 3]

$= \left(\frac{4-2}{8-3(2)+5}\right)^2$ [9, 7, and 8]

$= \left(\frac{2}{7}\right)^2 = \frac{4}{49}$

12. $\lim_{x\to 4}\frac{x^2-4x}{x^2-3x-4} = \lim_{x\to 4}\frac{x(x-4)}{(x-4)(x+1)} = \lim_{x\to 4}\frac{x}{x+1} = \frac{4}{4+1} = \frac{4}{5}$

18. $\lim_{h\to 0}\frac{(2+h)^3-8}{h} = \lim_{h\to 0}\frac{\left(8+12h+6h^2+h^3\right)-8}{h} = \lim_{h\to 0}\frac{12h+6h^2+h^3}{h}$

$= \lim_{h\to 0}\left(12+6h+h^2\right) = 12+0+0 = 12$

24. $\lim_{x\to -1}\frac{x^2+2x+1}{x^4-1} = \lim_{x\to -1}\frac{(x+1)^2}{(x^2+1)(x^2-1)} = \lim_{x\to -1}\frac{(x+1)^2}{(x^2+1)(x+1)(x-1)}$

$= \lim_{x\to -1}\frac{x+1}{(x^2+1)(x-1)} = \frac{0}{2(-2)} = 0$

28. $\lim_{h\to 0}\frac{(3+h)^{-1}-3^{-1}}{h} = \lim_{h\to 0}\frac{\frac{1}{3+h}-\frac{1}{3}}{h} = \lim_{h\to 0}\frac{3-(3+h)}{h(3+h)3} = \lim_{h\to 0}\frac{-h}{h(3+h)3}$

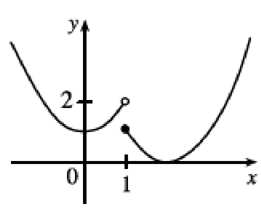
$= \lim_{h\to 0}\left[-\frac{1}{3(3+h)}\right] = -\frac{1}{\lim_{h\to 0}[3(3+h)]} = -\frac{1}{3(3+0)} = -\frac{1}{9}$

48. (a) $f(x) = \begin{cases} x^2+1 & \text{if } x<1 \\ (x-2)^2 & \text{if } x\geq 1 \end{cases}$

$\lim_{x\to 1^-}f(x) = \lim_{x\to 1^-}(x^2+1) = 1^2+1 = 2,\quad \lim_{x\to 1^+}f(x) = \lim_{x\to 1^+}(x-2)^2 = (-1)^2 = 1$

(b) Since the right-hand and left-hand limits of f at $x=1$ are not equal, $\lim_{x\to 1}f(x)$ does not exist.

(c)



64. *Solution 1:* First, we find the coordinates of P and Q as functions of r. Then we can find the equation of the line determined by these two points, and thus find the x-intercept (the point R), and take the limit as $r \to 0$. The coordinates of P are $(0, r)$. The point Q is the point of intersection of the two circles $x^2 + y^2 = r^2$ and $(x-1)^2 + y^2 = 1$. Eliminating y from these equations, we get $r^2 - x^2 = 1 - (x-1)^2 \quad \Leftrightarrow \quad r^2 = 1 + 2x - 1 \quad \Leftrightarrow \quad x = \frac{1}{2}r^2$. Substituting back into the equation of the shrinking circle to find the y-coordinate, we get $\left(\frac{1}{2}r^2\right)^2 + y^2 = r^2 \quad \Leftrightarrow \quad y^2 = r^2\left(1 - \frac{1}{4}r^2\right) \quad \Leftrightarrow \quad y = r\sqrt{1 - \frac{1}{4}r^2}$ (the positive y-value). So the coordinates of Q are $\left(\frac{1}{2}r^2, r\sqrt{1 - \frac{1}{4}r^2}\right)$. The equation of the line joining P and Q is thus $y - r = \dfrac{r\sqrt{1 - \frac{1}{4}r^2} - r}{\frac{1}{2}r^2 - 0}(x - 0)$. We set $y = 0$ in order to find the x-intercept, and get

$$x = -r\frac{\frac{1}{2}r^2}{r\left(\sqrt{1 - \frac{1}{4}r^2} - 1\right)} = \frac{-\frac{1}{2}r^2\left(\sqrt{1 - \frac{1}{4}r^2} + 1\right)}{1 - \frac{1}{4}r^2 - 1} = 2\left(\sqrt{1 - \frac{1}{4}r^2} + 1\right)$$

Now we take the limit as $r \to 0^+$: $\lim\limits_{r \to 0^+} x = \lim\limits_{r \to 0^+} 2\left(\sqrt{1 - \frac{1}{4}r^2} + 1\right) = \lim\limits_{r \to 0^+} 2\left(\sqrt{1} + 1\right) = 4$.

So the limiting position of R is the point $(4, 0)$.

Solution 2: We add a few lines to the diagram, as shown. Note that $\angle PQS = 90°$ (subtended by diameter PS). So $\angle SQR = 90° = \angle OQT$ (subtended by diameter OT). It follows that $\angle OQS = \angle TQR$. Also $\angle PSQ = 90° - \angle SPQ = \angle ORP$. Since $\triangle QOS$ is isosceles, so is $\triangle QTR$, implying that $QT = TR$. As the circle C_2 shrinks, the point Q plainly approaches the origin, so the point R must approach a point twice as far from the origin as T, that is, the point $(4, 0)$, as above.