6. $\lim_{x\to 1^+} \dfrac{x^2-9}{x^2+2x-3} = -\infty$ since $x^2+2x-3 \to 0^+$ as $x \to 1^+$ and $\dfrac{x^2-9}{x^2+2x-3} < 0$ for $1 < x < 3$.

14. Since x is negative, $\sqrt{x^2} = |x| = -x$. Thus,

$$\lim_{x\to-\infty} \frac{\sqrt{x^2-9}}{2x-6} = \lim_{x\to-\infty} \frac{\sqrt{x^2-9}/\sqrt{x^2}}{(2x-6)/(-x)} = \lim_{x\to-\infty} \frac{\sqrt{1-9/x^2}}{-2+6/x} = \frac{\sqrt{1-0}}{-2+0} = -\frac{1}{2}$$

31. $\sin x$ and e^x are continuous on $\mathbb{R}$ by Theorem 7 in Section 2.5. Since e^x is continuous on $\mathbb{R}$, $e^{\sin x}$ is continuous on $\mathbb{R}$ by Theorem 9 in Section 2.5. Lastly, x is continuous on $\mathbb{R}$ since it's a polynomial and the product $xe^{\sin x}$ is continuous on its domain $\mathbb{R}$ by Theorem 4 in Section 2.5.

38. (a) When V increases from 200 in^3 to 250 in^3, we have $\Delta V = 250 - 200 = 50$ in^3, and since $P = 800/V$,

$\Delta P = P(250) - P(200) = \dfrac{800}{250} - \dfrac{800}{200} = 3.2 - 4 = -0.8$ lb/in^2. So the average rate of change

is $\dfrac{\Delta P}{\Delta V} = \dfrac{-0.8}{50} = -0.016\ \dfrac{\text{lb/in}^2}{\text{in}^3}$.

(b) Since $V = 800/P$, the instantaneous rate of change of V with respect to P is

$$\lim_{h\to0} \frac{\Delta V}{\Delta P} = \lim_{h\to0} \frac{V(P+h) - V(P)}{h} = \lim_{h\to0} \frac{800/(P+h) - 800/P}{h} = \lim_{h\to0} \frac{800\,[P-(P+h)]}{h(P+h)P}$$
$$= \lim_{h\to0} \frac{-800}{(P+h)P} = -\frac{800}{P^2}$$

which is inversely proportional to the square of P.

48. The graph of a has tangent lines with positive slope for $x < 0$ and negative slope for $x > 0$, and the values of c fit this pattern, so c must be the graph of the derivative of the function for a. The graph of c has horizontal tangent lines to the left and right of the x-axis and b has zeros at these points. Hence, b is the graph of the derivative of the function for c. Therefore, a is the graph of f, c is the graph of f', and b is the graph of f''.

14. $y = \ln \sec x \quad \Rightarrow \quad y' = \dfrac{1}{\sec x}\dfrac{d}{dx}(\sec x) = \dfrac{1}{\sec x}(\sec x \tan x) = \tan x$

30. $y = \dfrac{(x^2+1)^4}{(2x+1)^3(3x-1)^5} \quad \Rightarrow$

$$\ln y = \ln \frac{(x^2+1)^4}{(2x+1)^3(3x-1)^5} = \ln(x^2+1)^4 - \ln[(2x+1)^3(3x-1)^5] = 4\ln(x^2+1) - [\ln(2x+1)^3 + \ln(3x-1)^5]$$

$$= 4\ln(x^2+1) - 3\ln(2x+1) - 5\ln(3x-1) \quad \Rightarrow$$

$$\frac{y'}{y} = 4 \cdot \frac{1}{x^2+1} \cdot 2x - 3 \cdot \frac{1}{2x+1} \cdot 2 - 5 \cdot \frac{1}{3x-1} \cdot 3 \quad \Rightarrow \quad y' = \frac{(x^2+1)^4}{(2x+1)^3(3x-1)^5}\left(\frac{8x}{x^2+1} - \frac{6}{2x+1} - \frac{15}{3x-1}\right).$$

[The answer could be simplified to $y' = -\dfrac{(x^2+56x+9)(x^2+1)^3}{(2x+1)^4(3x-1)^6}$, but this is unnecessary.]

47. $y = \cosh^{-1}(\sinh x) \quad \Rightarrow \quad y' = \dfrac{1}{\sqrt{(\sinh x)^2 - 1}} \cdot \cosh x = \dfrac{\cosh x}{\sqrt{\sinh^2 x - 1}}$

61. $y = (2+x)e^{-x} \quad \Rightarrow \quad y' = (2+x)(-e^{-x}) + e^{-x} \cdot 1 = e^{-x}[-(2+x)+1] = e^{-x}(-x-1)$.

At $(0,2)$, $y' = 1(-1) = -1$, so an equation of the tangent line is $y - 2 = -1(x-0)$, or $y = -x+2$.

The slope of the normal line is 1, so an equation of the normal line is $y - 2 = 1(x-0)$, or $y = x+2$.

69. (a) $h(x) = f(x)\,g(x) \quad \Rightarrow \quad h'(x) = f(x)\,g'(x) + g(x)\,f'(x) \quad \Rightarrow$

$h'(2) = f(2)\,g'(2) + g(2)\,f'(2) = (3)(4) + (5)(-2) = 12 - 10 = 2$

(b) $F(x) = f(g(x)) \quad \Rightarrow \quad F'(x) = f'(g(x))\,g'(x) \quad \Rightarrow \quad F'(2) = f'(g(2))\,g'(2) = f'(5)(4) = 11 \cdot 4 = 44$

74. $f(x) = g(g(x)) \quad \Rightarrow \quad f'(x) = g'(g(x))\,g'(x)$

85. $y = f(x) = ax^2 + bx + c \quad \Rightarrow \quad f'(x) = 2ax + b$. We know that $f'(-1) = 6$ and $f'(5) = -2$, so $-2a + b = 6$ and $10a + b = -2$. Subtracting the first equation from the second gives $12a = -8 \quad \Rightarrow \quad a = -\frac{2}{3}$. Substituting $-\frac{2}{3}$ for a in the first equation gives $b = \frac{14}{3}$. Now $f(1) = 4 \quad \Rightarrow \quad 4 = a + b + c$, so $c = 4 + \frac{2}{3} - \frac{14}{3} = 0$ and hence, $f(x) = -\frac{2}{3}x^2 + \frac{14}{3}x$.

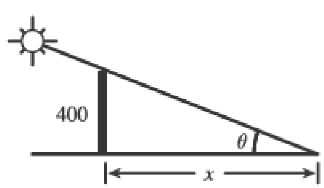
97. If $x =$ edge length, then $V = x^3 \quad \Rightarrow \quad dV/dt = 3x^2\,dx/dt = 10 \quad \Rightarrow \quad dx/dt = 10/(3x^2)$ and $S = 6x^2 \quad \Rightarrow$

$dS/dt = (12x)\,dx/dt = 12x[10/(3x^2)] = 40/x$. When $x = 30$, $dS/dt = \frac{40}{30} = \frac{4}{3}\ \text{cm}^2/\text{min}$.

101. We are given $d\theta/dt = -0.25$ rad/h. $\tan\theta = 400/x \quad \Rightarrow$

$x = 400\cot\theta \quad \Rightarrow \quad \dfrac{dx}{dt} = -400\csc^2\theta\,\dfrac{d\theta}{dt}$. When $\theta = \frac{\pi}{6}$,

$\dfrac{dx}{dt} = -400(2)^2(-0.25) = 400$ ft/h.

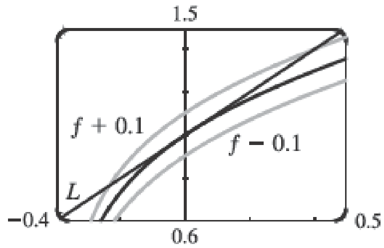


103. (a) $f(x) = \sqrt[3]{1+3x} = (1+3x)^{1/3} \quad \Rightarrow \quad f'(x) = (1+3x)^{-2/3}$, so the linearization of f at $a = 0$ is

$L(x) = f(0) + f'(0)(x-0) = 1^{1/3} + 1^{-2/3}x = 1 + x$. Thus, $\sqrt[3]{1+3x} \approx 1 + x \quad \Rightarrow$

$\sqrt[3]{1.03} = \sqrt[3]{1+3(0.01)} \approx 1 + (0.01) = 1.01$.

(b) The linear approximation is $\sqrt[3]{1+3x} \approx 1 + x$, so for the required accuracy we want $\sqrt[3]{1+3x} - 0.1 < 1 + x < \sqrt[3]{1+3x} + 0.1$. From the graph, it appears that this is true when $-0.235 < x < 0.401$.



5. $f(x) = x + 2\cos x$, $[-\pi, \pi]$. $f'(x) = 1 - 2\sin x$. $f'(x) = 0 \quad \Rightarrow \quad \sin x = \frac{1}{2} \quad \Rightarrow \quad x = \frac{\pi}{6}, \frac{5\pi}{6}$. $f'(x) > 0$ for $\left(-\pi, \frac{\pi}{6}\right)$ and $\left(\frac{5\pi}{6}, \pi\right)$, and $f'(x) < 0$ for $\left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$, so $f\left(\frac{\pi}{6}\right) = \frac{\pi}{6} + \sqrt{3} \approx 2.26$ is a local maximum value and $f\left(\frac{5\pi}{6}\right) = \frac{5\pi}{6} - \sqrt{3} \approx 0.89$ is a local minimum value. Checking the endpoints, we find $f(-\pi) = -\pi - 2 \approx -5.14$ and $f(\pi) = \pi - 2 \approx 1.14$. Thus, $f(-\pi) = -\pi - 2$ is the absolute minimum value and $f\left(\frac{\pi}{6}\right) = \frac{\pi}{6} + \sqrt{3}$ is the absolute maximum value.

10. This limit has the form $\frac{\infty}{\infty}$. $\displaystyle\lim_{x\to\infty} \frac{e^{4x} - 1 - 4x}{x^2} \overset{\text{H}}{=} \lim_{x\to\infty} \frac{4e^{4x} - 4}{2x} \overset{\text{H}}{=} \lim_{x\to\infty} \frac{16e^{4x}}{2} = \lim_{x\to\infty} 8e^{4x} = \infty$

22. $y = f(x) = \dfrac{x}{1-x^2}$ **A.** $D = (-\infty, -1) \cup (-1, 1) \cup (1, \infty)$ **B.** y-intercept: $f(0) = 0$; x-intercept: 0

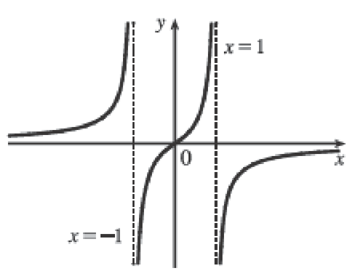
C. $f(-x) = -f(x)$, so f is odd and the graph is symmetric about the origin. **D.** $\displaystyle\lim_{x\to\pm\infty} \frac{x}{1-x^2} = 0$, so $y = 0$ is a HA.

$\displaystyle\lim_{x\to-1^-} \frac{x}{1-x^2} = \infty$ and $\displaystyle\lim_{x\to-1^+} \frac{x}{1-x^2} = -\infty$, so $x = -1$ is a VA. Similarly, $\displaystyle\lim_{x\to1^-} \frac{x}{1-x^2} = \infty$ and $\displaystyle\lim_{x\to1^+} \frac{x}{1-x^2} = -\infty$, so $x = 1$ is a VA. **E.** $f'(x) = \dfrac{(1-x^2)(1) - x(-2x)}{(1-x^2)^2} = \dfrac{1+x^2}{(1-x^2)^2} > 0$ for $x \neq \pm 1$, so f is increasing on $(-\infty, -1)$, $(-1, 1)$, and $(1, \infty)$. **F.** No local extrema

G. $$f''(x) = \frac{(1-x^2)^2(2x) - (1+x^2)2(1-x^2)(-2x)}{[(1-x^2)^2]^2} = \frac{2x(1-x^2)[(1-x^2) + 2(1+x^2)]}{(1-x^2)^4} = \frac{2x(3+x^2)}{(1-x^2)^3}$$

$f''(x) > 0$ for $x < -1$ and $0 < x < 1$, and $f''(x) < 0$ for $-1 < x < 0$ and $x > 1$, so f is CU on $(-\infty, -1)$ and $(0, 1)$, and f is CD on $(-1, 0)$ and $(1, \infty)$. $(0, 0)$ is an IP.

H.



26. $y = f(x) = \sqrt{1-x} + \sqrt{1+x}$ **A.** $1 - x \geq 0$ and $1 + x \geq 0 \Rightarrow x \leq 1$ and $x \geq -1$, so $D = [-1, 1]$.

B. y-intercept: $f(0) = 1 + 1 = 2$; no x-intercept because $f(x) > 0$ for all x.

C. $f(-x) = f(x)$, so the curve is symmetric about the y-axis. **D.** No asymptote

E. $f'(x) = \frac{1}{2}(1-x)^{-1/2}(-1) + \frac{1}{2}(1+x)^{-1/2} = \dfrac{-1}{2\sqrt{1-x}} + \dfrac{1}{2\sqrt{1+x}} = \dfrac{-\sqrt{1+x}+\sqrt{1-x}}{2\sqrt{1-x}\sqrt{1+x}} > 0 \Rightarrow$

$-\sqrt{1+x} + \sqrt{1-x} > 0 \Rightarrow \sqrt{1-x} > \sqrt{1+x} \Rightarrow 1 - x > 1 + x \Rightarrow -2x > 0 \Rightarrow x < 0$, so $f'(x) > 0$ for

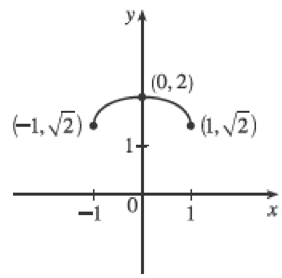
$-1 < x < 0$ and $f'(x) < 0$ for $0 < x < 1$. Thus, f is increasing on $(-1, 0)$ and decreasing on $(0, 1)$. **F.** Local maximum value $f(0) = 2$

G. $f''(x) = -\frac{1}{2}\left(-\frac{1}{2}\right)(1-x)^{-3/2}(-1) + \frac{1}{2}\left(-\frac{1}{2}\right)(1+x)^{-3/2}$

$$= \frac{-1}{4(1-x)^{3/2}} + \frac{-1}{4(1+x)^{3/2}} < 0$$

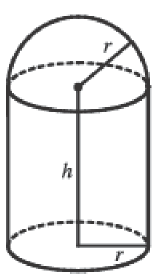
for all x in the domain, so f is CD on $(-1, 1)$. No IP

H.



47. Since f is continuous on $[32, 33]$ and differentiable on $(32, 33)$, then by the Mean Value Theorem there exists a number c in $(32, 33)$ such that $f'(c) = \frac{1}{5}c^{-4/5} = \dfrac{\sqrt[5]{33} - \sqrt[5]{32}}{33 - 32} = \sqrt[5]{33} - 2$, but $\frac{1}{5}c^{-4/5} > 0 \Rightarrow \sqrt[5]{33} - 2 > 0 \Rightarrow \sqrt[5]{33} > 2$. Also f' is decreasing, so that $f'(c) < f'(32) = \frac{1}{5}(32)^{-4/5} = 0.0125 \Rightarrow 0.0125 > f'(c) = \sqrt[5]{33} - 2 \Rightarrow \sqrt[5]{33} < 2.0125$. Therefore, $2 < \sqrt[5]{33} < 2.0125$.

58.



We minimize the surface area $S = \pi r^2 + 2\pi rh + \frac{1}{2}(4\pi r^2) = 3\pi r^2 + 2\pi rh$.

Solving $V = \pi r^2 h + \frac{2}{3}\pi r^3$ for h, we get $h = \dfrac{V - \frac{2}{3}\pi r^3}{\pi r^2} = \dfrac{V}{\pi r^2} - \frac{2}{3}r$, so

$S(r) = 3\pi r^2 + 2\pi r\left[\dfrac{V}{\pi r^2} - \frac{2}{3}r\right] = \frac{5}{3}\pi r^2 + \dfrac{2V}{r}$.

$S'(r) = -\dfrac{2V}{r^2} + \frac{10}{3}\pi r = \dfrac{\frac{10}{3}\pi r^3 - 2V}{r^2} = 0 \Leftrightarrow \frac{10}{3}\pi r^3 = 2V \Leftrightarrow r^3 = \dfrac{3V}{5\pi} \Leftrightarrow r = \sqrt[3]{\dfrac{3V}{5\pi}}$.

This gives an absolute minimum since $S'(r) < 0$ for $0 < r < \sqrt[3]{\dfrac{3V}{5\pi}}$ and $S'(r) > 0$ for $r > \sqrt[3]{\dfrac{3V}{5\pi}}$. Thus,

$$h = \frac{V - \frac{2}{3}\pi \cdot \frac{3V}{5\pi}}{\pi\sqrt[3]{\frac{(3V)^2}{(5\pi)^2}}} = \frac{\left(V - \frac{2}{5}V\right)\sqrt[3]{(5\pi)^2}}{\pi\sqrt[3]{(3V)^2}} = \frac{3V\sqrt[3]{(5\pi)^2}}{5\pi\sqrt[3]{(3V)^2}} = \sqrt[3]{\frac{3V}{5\pi}} = r$$

59. Let x denote the number of \$1 decreases in ticket price. Then the ticket price is $\$12 - \$1(x)$, and the average attendance is $11{,}000 + 1000(x)$. Now the revenue per game is

$$\begin{aligned} R(x) &= (\text{price per person}) \times (\text{number of people per game}) \\ &= (12 - x)(11{,}000 + 1000x) = -1000x^2 + 1000x + 132{,}000 \end{aligned}$$

for $0 \leq x \leq 4$ [since the seating capacity is 15,000] $\Rightarrow \quad R'(x) = -2000x + 1000 = 0 \quad \Leftrightarrow \quad x = 0.5$. This is a maximum since $R''(x) = -2000 < 0$ for all x. Now we must check the value of $R(x) = (12 - x)(11{,}000 + 1000x)$ at $x = 0.5$ and at the endpoints of the domain to see which value of x gives the maximum value of R.

$R(0) = (12)(11{,}000) = 132{,}000$, $R(0.5) = (11.5)(11{,}500) = 132{,}250$, and $R(4) = (8)(15{,}000) = 120{,}000$. Thus, the maximum revenue of \$132,250 per game occurs when the average attendance is 11,500 and the ticket price is \$11.50.

69. $f'(t) = 2t - 3\sin t \quad \Rightarrow \quad f(t) = t^2 + 3\cos t + C$.

$f(0) = 3 + C$ and $f(0) = 5 \quad \Rightarrow \quad C = 2$, so $f(t) = t^2 + 3\cos t + 2$.

4. On $[0, \pi]$, $\lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \sin x_i \, \Delta x = \int_0^\pi \sin x \, dx = [-\cos x]_0^\pi = -(-1) - (-1) = 2$.

8. (a) By the Net Change Theorem (FTC2), $\displaystyle\int_0^1 \frac{d}{dx}\left(e^{\arctan x}\right) dx = \left[e^{\arctan x}\right]_0^1 = e^{\pi/4} - 1$

(b) $\displaystyle\frac{d}{dx} \int_0^1 e^{\arctan x} \, dx = 0$ since this is the derivative of a constant.

(c) By FTC1, $\displaystyle\frac{d}{dx} \int_0^x e^{\arctan t} \, dt = e^{\arctan x}$.

12. Let $u = 1 - x$, so $du = -dx$ and $dx = -du$. When $x = 0$, $u = 1$; when $x = 1$, $u = 0$. Thus,

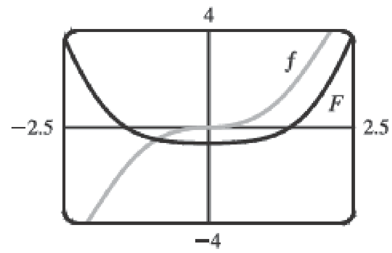
$\int_0^1 (1 - x)^9 \, dx = \int_1^0 u^9(-du) = \int_0^1 u^9 \, du = \frac{1}{10}\left[u^{10}\right]_0^1 = \frac{1}{10}(1 - 0) = \frac{1}{10}$.

20. $\displaystyle\int_{-1}^{1} \frac{\sin x}{1 + x^2} \, dx = 0$ by Theorem 5.5.7(b), since $f(x) = \dfrac{\sin x}{1 + x^2}$ is an odd function.

32. Let $u = x^2$. Then $du = 2x \, dx$, so $\displaystyle\int \frac{x}{\sqrt{1 - x^4}} \, dx = \frac{1}{2} \int \frac{du}{\sqrt{1 - u^2}} = \tfrac{1}{2}\sin^{-1} u + C = \tfrac{1}{2}\sin^{-1}\left(x^2\right) + C$.

40. Let $u = x^2 + 1$. Then $x^2 = u - 1$ and $x\,dx = \frac{1}{2}\,du$, so

$$\int \frac{x^3}{\sqrt{x^2+1}}\,dx = \int \frac{(u-1)}{\sqrt{u}}\left(\tfrac{1}{2}\,du\right) = \frac{1}{2}\int \left(u^{1/2} - u^{-1/2}\right)du$$
$$= \tfrac{1}{2}\left(\tfrac{2}{3}u^{3/2} - 2u^{1/2}\right) + C = \tfrac{1}{3}(x^2+1)^{3/2} - (x^2+1)^{1/2} + C$$
$$= \tfrac{1}{3}(x^2+1)^{1/2}\left[(x^2+1) - 3\right] + C = \tfrac{1}{3}\sqrt{x^2+1}\,(x^2-2) + C$$



48. $y = \int_{2x}^{3x+1} \sin(t^4)\,dt = \int_{2x}^{0} \sin(t^4)\,dt + \int_{0}^{3x+1} \sin(t^4)\,dt = \int_{0}^{3x+1} \sin(t^4)\,dt - \int_{0}^{2x} \sin(t^4)\,dt \quad \Rightarrow$

$y' = \sin\left[(3x+1)^4\right] \cdot \frac{d}{dx}(3x+1) - \sin\left[(2x)^4\right] \cdot \frac{d}{dx}(2x) = 3\sin\left[(3x+1)^4\right] - 2\sin\left[(2x)^4\right]$

58. Distance covered $= \int_0^{5.0} v(t)\,dt \approx M_5 = \frac{5.0-0}{5}\left[v(0.5) + v(1.5) + v(2.5) + v(3.5) + v(4.5)\right]$

$= 1(4.67 + 8.86 + 10.22 + 10.67 + 10.81) = 45.23\text{ m}$

67. Let $u = f(x)$ and $du = f'(x)\,dx$. So $2\int_a^b f(x)f'(x)\,dx = 2\int_{f(a)}^{f(b)} u\,du = \left[u^2\right]_{f(a)}^{f(b)} = [f(b)]^2 - [f(a)]^2$.

6. $\displaystyle\int_1^2 x^5 \ln x\,dx = \left[\tfrac{1}{6}x^6 \ln x\right]_1^2 - \int_1^2 \tfrac{1}{6}x^5\,dx \qquad \begin{bmatrix} u = \ln x, & dv = x^5\,dx \\ du = \frac{1}{x}\,dx, & v = \frac{1}{6}x^6 \end{bmatrix}$

$= \frac{64}{6}\ln 2 - 0 - \left[\frac{1}{36}x^6\right]_1^2 = \frac{32}{3}\ln 2 - \left(\frac{64}{36} - \frac{1}{36}\right) = \frac{32}{3}\ln 2 - \frac{7}{4}$

12. $\displaystyle\int \frac{e^{2x}}{1+e^{4x}}\,dx = \int \frac{1}{1+u^2}\left(\tfrac{1}{2}\,du\right) \qquad \begin{bmatrix} u = e^{2x}, \\ du = 2e^{2x}\,dx \end{bmatrix}$

$= \frac{1}{2}\tan^{-1} u + C = \frac{1}{2}\tan^{-1} e^{2x} + C$

24. Let $u = \cos x$, $dv = e^x\,dx \quad \Rightarrow \quad du = -\sin x\,dx$, $v = e^x$: $(*)\ I = \int e^x \cos x\,dx = e^x \cos x + \int e^x \sin x\,dx$.

To integrate $\int e^x \sin x\,dx$, let $U = \sin x$, $dV = e^x\,dx \quad \Rightarrow \quad dU = \cos x\,dx$, $V = e^x$. Then

$\int e^x \sin x\,dx = e^x \sin x - \int e^x \cos x\,dx = e^x \sin x - I$. By substitution in $(*)$, $I = e^x \cos x + e^x \sin x - I \quad \Rightarrow$

$2I = e^x(\cos x + \sin x) \quad \Rightarrow \quad I = \frac{1}{2}e^x(\cos x + \sin x) + C$.