Discrete Math Homework Set 2
Due July 7th

1. Please provide brief justification for your calculations.

(i) Find the smallest positive integer x satisfying

$$8 \cdot 9 \cdot 10 \cdot 11 \cdot 12 \cdot 13 \equiv x \mod 7.$$

(ii) Find the smallest positive integer x satisfying

$$3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8 \cdot 9 \cdot 10 \equiv x \mod 11.$$

(iii) Find the remainder when 3^{100} is divided by 7.

(iv) Use congruence mod 9 to find the missing digit ? in

$$(92854) \cdot (16284) = 151?034536.$$

2. Prove by induction that for any natural number n

$$\sum_{i=1}^{n} (2i-1)^2 = \frac{n(2n-1)(2n+1)}{3}.$$

3. Let S be the set of all ordered pairs of positive real numbers (x, y) and let T be the set of all ordered pairs of positive real numbers (a, b) such that $ab < 1$. Show that $f(x, y) = (\frac{x}{y}, \frac{y}{x+1})$ defines a bijection from S to T.

4. (i) Find all integers x, y such that

$$(3x + y \equiv 1 \mod 9) \text{ and } (2x - y \equiv 0 \mod 9).$$

(ii) Explain why there are no integers x, y satisfying

$$(3x + y \equiv 1 \mod 5) \text{ and } (2x - y \equiv 0 \mod 5).$$

5. A President, Treasurer, and Secretary, all different, are to be chosen from a club consisting of 10 people. How many different choices of officers are possible if

(i) there are no restrictions;

(ii) A and B will not serve together;

(iii) C and D will serve together or not at all;

(iv) E must be an officer;

(v) F will serve only if she is president?

Bonus How many numbers in the sequence $101, 10101, 1010101, \ldots$ are prime?