

Discrete Math Homework Set 1

Due June 30th

1. You have 12 pencils which all look the same. Eleven of the pencils have the same weight and the other is either heavier or lighter (you don't know) than these eleven. You have a balance and want to determine the odd pencil by using the balance 3 times. You begin by randomly choosing 8 pencils and putting 4 on each side of the balance.

(i) Assume that the pencils balance. Show how to use the balance 2 more times to determine which is the odd pencil and whether it is heavier or lighter than the rest. Note: Clearly state your assumptions, logic and conclusions.

(ii) (Bonus) Assume that the two sets of 4 pencils do not balance. Show how to use the balance 2 more times to determine which is the odd pencil and whether it is heavier or lighter than the rest.

2. It is dark outside and 4 people are on one side of a river. There is a bridge across the river which can hold at most two people at a time. The people have one flashlight and a person or pair always needs to have it when crossing the bridge. Person A can cross the bridge in 1 minute if walking alone, person B can cross the bridge in 2 minutes if walking alone, person C can cross the bridge in 5 minutes if walking alone and person D can cross the bridge in 10 minutes if walking alone. Given that a pair of people must walk at the slower person's rate when crossing the bridge, what is the minimum amount of time necessary for all four people to cross to the other side? Give an argument convincing us that your solution is optimal. Note: There is no trick here, the people don't swim across or throw the flashlight back and forth.

3. Show that for every natural number n

$$1^2 + 2^2 + 3^2 + 4^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}.$$

4. Show that $\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \dots + \frac{1}{\sqrt{n}} \geq \sqrt{n}$ for all natural numbers n .