SYLLABUS FOR 21-242 MATRIX THEORY

JAMES CUMMINGS

Course description: Matrix theory 21-242 is an honours course in elementary linear algebra. Linear algebra is the theory of vectors, matrices and linear transformations. Linear algebra is a key tool in many parts of pure and applied mathematics. It is used in geometry, analysis, particle physics, statistics, and optimisation to name but a few of its areas of application.

The course will emphasise the theoretical and conceptual aspects of the subject, and students will be expected to write proofs. The pace will be quite fast. It is particularly important that students attempt all the homework exercises since this is the best way to ensure a firm grasp on the concepts. We will use the Maple computer algebra system to work with examples that are too large to compute by hand.

Instructor: James Cummings

Office hours: 12:30-1:30 MWF or by appointment. Send me email at `jcumming@andrew.cmu.edu` to make an appointment.

Homework: Homework will generally be set each Wednesday and due in class the following Wednesday. Late homework will not be accepted under *any* circumstances. I will drop the two lowest homework scores.

Web page: There will be a web page for the course linked from
`www.math.cmu.edu/users/jcumming/teaching`
Homework and solutions will be posted here along with course announcements.

Class will meet MWF 11:30-12:20 in Porter Hall A18B.

The recitation instructor is Paul McKenney (`pmckenne@andrew.cmu.edu`). Recitation meets T 10:30-11:20 in Wean Hall 5409.

Exams and grading: There will be a midterm and a final, dates to be determined. Grades will be determined by a formula in which homework counts 30 percent, the midterm counts 25 percent and the final counts 45 percent.

Textbook: Hoffman and Kunze, "Linear algebra".

Supplementary reading: Halmos "Finite dimensional vector spaces", Lax "Linear algebra", Curtis "Linear Algebra: An Introductory Approach".

Course outline (tentative and subject to change):

- Writing mathematical proofs.
- The real numbers.
- Simultaneous linear equations.
- Matrices and vectors. Writing linear equations in matrix form.
- The arithmetic of vectors and matrices.
- Linear combination of vectors, span of a set of vectors, linear dependence.
- Linear transformations.
- Subspaces of $\mathbb{R}^n$. Bases and dimension of a subspace.
- Rowspace, columnspace and nullspace of a matrix.
- Rank and nullity of a matrix. The rank-nullity theorem.
- Row and column operations. Linear equations revisited
- Inverse of a matrix.
- Determinants.
- Change of basis in $\mathbb{R}^n$.
- Characteristic equation, eigenvalues and eigenvectors, geometric and algebraic multiplicty of an eigenvalue.
- Inner product in $\mathbb{R}^n$, orthonormal bases, Gram-Schmidt process.
- Symmetric and orthogonal matrices, diagonalisation of a symmetric matrix.
- Complex numbers.
- The Jordan and rational canonical forms.
- $\mathbb{C}^n$ as a complex vector space and a complex inner product space.
- Hermitian and unitary matrices.
- Quadratic forms. Law of inertia. Positive definite and positive semidefinite matrices.