MATRIX THEORY AUTUMN 2011: HOMEWORK 3 SOLUTIONS

JAMES CUMMINGS

Homework due at start of class on Wed 14 September.
Page 10 Q2: I am using the algorithm which is implict in the proof of Theorem 4.

$$A_0 = A = \begin{pmatrix} 3 & -1 & 2 \\ 2 & 1 & 1 \\ 1 & -3 & 0 \end{pmatrix}$$

Multiply row 1 by 1/3.

$$A_1 = \begin{pmatrix} 1 & -1/3 & 2/3 \\ 2 & 1 & 1 \\ 1 & -3 & 0 \end{pmatrix}$$

Add -2 times row 1 to row 2.

$$A_2 = \begin{pmatrix} 1 & -1/3 & 2/3 \\ 0 & 5/3 & -1/3 \\ 1 & -3 & 0 \end{pmatrix}$$

Add -1 times row 1 to row 3.

$$A_3 = \begin{pmatrix} 1 & -1/3 & 2/3 \\ 0 & 5/3 & -1/3 \\ 0 & -8/3 & -2/3 \end{pmatrix}$$

Multiply row 2 by 3/5.

$$A_4 = \begin{pmatrix} 1 & -1/3 & 2/3 \\ 0 & 1 & -1/5 \\ 0 & -8/3 & -2/3 \end{pmatrix}$$

Add 1/3 times row 2 to row 1

$$A_5 = \begin{pmatrix} 1 & 0 & 3/5 \\ 0 & 1 & -1/5 \\ 0 & -8/3 & -2/3 \end{pmatrix}$$

Add 8/3 times row 2 to row 3.

$$A_6 = \begin{pmatrix} 1 & 0 & 3/5 \\ 0 & 1 & -1/5 \\ 0 & 0 & -6/5 \end{pmatrix}$$

Multiply row 3 by -5/6.

$$A_7 = \begin{pmatrix} 1 & 0 & 3/5 \\ 0 & 1 & -1/5 \\ 0 & 0 & 1 \end{pmatrix}$$

Add -3/5 times row 3 to row 1.

$$A_8 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -1/5 \\ 0 & 0 & 1 \end{pmatrix}$$

Add 1/5 times row 3 to row 2.

$$A_9 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Now $AX = 0$ if and only if $1X = X = 0$, and we are done.

Page 11 Q3: There are several ways to do this.

$AX = 2X$ if and only if $(A - 21_{3\times 3})X = 0$.

$$A - 21_{3\times 3} = \begin{pmatrix} 4 & -4 & 0 \\ 4 & -4 & 0 \\ -1 & 0 & 1 \end{pmatrix}$$

Putting in reduced row echelon form gives

$$\begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{pmatrix}$$

So the general solution is $x_1 = x_2 = x_3$, to put it another way the solutions are the multiples of the column vector

$$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$A - 31_{3\times 3} = \begin{pmatrix} 3 & -4 & 0 \\ 4 & -5 & 0 \\ -1 & 0 & 0 \end{pmatrix}$$

By row reducing it is easy to see that any $(x_1, x_2, x_3) \in \mathbb{R}^3$ with $x_1 = x_2 = 0$ is a solution.

Page 11 Q5: Call the two matrices A and B. If they were row equivalent the equations $AX = 0$ and $BX = 0$ would have the same solutions. An easy row reduction calculation shows that A is row equivalent to 1, so $AX = 0$ has only $X = 0$ as a solution. However another easy row reduction calculation shows that $BX = 0$ has

$$\begin{pmatrix} -1 \\ -3 \\ 2 \end{pmatrix}$$

as a solution.

Page 11 Q7: Let A be $m \times n$ and let $a \neq b$ with $1 \leq a, b \leq m$. Consider the following sequence of row operations:

Add -1 times row a to row b, add row b to row a, multiply row b by -1, add row a to row b.

(Can you do it in fewer moves?)

Page 16 Q3:

Case 1: All rows are zero, and it is the zero matrix

$$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

Case 2: One row is zero. It must be the second row. The leading entry of the first row either occurs in column one or column two. If it occurs in column one then the matrix is of the form

$$\begin{pmatrix} 1 & a \\ 0 & 0 \end{pmatrix}$$

and an such matrix works. If the leading entry is in the second column then

$$\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

is the only possibility.

Case 3: Both rows are non-zero, the leading entry of column i is in row i and so

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

is the only possibility.