SYLLABUS FOR 21-374 FIELD THEORY

JAMES CUMMINGS

Instructor: James Cummings
Office hours: 1030-1130 MWF or by appointment. Send me email at jcumming@andrew.cmu.edu to make an appointment.

Homework: Homework will generally be set each Monday and due in class the following Monday. Late homework will not be accepted under any circumstances. I will drop the two lowest homework scores.

Web page: There will be a web page for the course linked from www.math.cmu.edu/users/jcumming/teaching. Homework and solutions will be posted here along with course announcements.

Exams and grading: There will be an in-class midterm and a take-home final, dates to be determined. Grades will be determined by a formula in which homework counts 30 percent, the midterm counts 25 percent and the final counts 45 percent.

Textbook: Dummit and Foote “Abstract algebra”.

Supplementary reading: Stewart “Galois theory”, Artin “Galois theory”.

Prerequisites: 21-373 Algebraic Structures or equivalent knowledge of the rudiments of groups, rings and fields. Ask me if you are in any doubt.

Course description: Fields are important structures in modern mathematics. You are probably already familiar with the rationals, the reals, and the complex numbers. Some other more exotic examples are the finite fields, the $p$-adic fields, the algebraic number fields and fields of rational functions.

We will study the structure of fields, with an emphasis on the analysis of field extensions. We will then use our knowledge of fields to prove several classical results in algebra and geometry. In particular...
we will prove the fundamental theorem of algebra, analyse which regular $n$-gons can be constructed with ruler and compasses, and discuss the solvability by radicals of polynomial equations with rational coefficients.

Course outline (tentative and subject to change)

- Review of background on rings, groups, fields and linear algebra.
- PIDs and UFDs.
- Euclidean domains.
- The fields $\mathbb{Q}$ and $\mathbb{Z}/p\mathbb{Z}$.
- Analysis of field extensions. Algebraic and transcendental elements. Degree of an extension.
- Ruler and compass constructions I: Squaring the circle and duplicating the cube.
- Adding a root. Existence and uniqueness of splitting fields.
- The Galois group. Galois extensions.
- The Galois correspondence. The fundamental theorem of Galois theory.
- Some easy applications. Fundamental theorem of algebra.
- Cyclotomy. Ruler and compass II: regular $n$-gons
- Algebraically closed fields and the algebraic closure.
- Finite fields.
- Radical extensions. Digression into group theory: solvable and simple groups, simplicity of $A_n$. Unsolvability of the quintic.
- Advanced topics (time allowing): A taste of algebraic number theory, $p$-adic fields, Galois theory for infinite degree extensions, Luroth’s theorem, separability revisited, skew fields, fields of rational functions, transcendence degree.