21-260 Differential Equations

Homework #1

Due Friday 20 January

1.3.2. Determine the order of the differential equation; also state whether the equation is linear or nonlinear.

\[(1 + y^2) \frac{dy}{dt} + t \frac{dy}{dt} + y = e^t.\]

1.3.10. Verify that each given function is a solution of the differential equation.

\[y'''' + 4y''' + 3y = t; \quad y_1(t) = \frac{t}{3}, \quad y_2(t) = e^{-t} + \frac{t}{3}\]

1.3.27. Verify that each given function is a solution of the given partial differential equation.

\[a^2 u_{xx} = u_{tt}; \quad u_1(x, t) = \sin \lambda x \sin \lambda at, \quad u_2(x, t) = \sin(x - at),\]

where \(\lambda\) is a real constant.

1.3.30. Another way to derive the pendulum equation

\[\frac{d^2 \theta}{dt^2} + \frac{g}{L} \sin \theta = 0 \quad (1)\]

is based on the principle of conservation of energy. Here \(L\) is the length of the pendulum, and \(\theta\) measures the angle the pendulum makes with a vertical line, measured from the downward position.

1. Show that the kinetic energy \(T\) of the pendulum in motion is

\[T = \frac{1}{2} mL^2 \left(\frac{d\theta}{dt}\right)^2.\]

2. Show that the potential energy \(V\) of the pendulum, relative to its rest position, is

\[V = mgL(1 - \cos \theta).\]

3. By the principle of conservation of energy, the total energy \(E = T + V\) is constant. Calculate \(dE/dt\), set it equal to zero, and show that the resulting equation reduces to Eq. (1).