Final Exam Review
Spring 2020

1. Evaluate the integrals

   (a) $\int \frac{16}{(x-3)(x+1)^2}\,dx$

   (b) $\int \frac{1}{x^2\sqrt{x^2+1}}\,dx$

2. Find the Taylor series for the function $f(x) = \frac{1}{x}$ centered at $a = 2$. What is the radius of convergence for this series? What is it's interval of convergence?

3. Consider the differential equation
$$\frac{dy}{dx} = \frac{y^2}{x-1}$$

   (a) Find all solutions to the differential equation. Solve explicitly for y as a function of x.

   (b) Is there a solution satisfying $y(0) = 0$? If so, find it. If not, explain why not.

4. (a) Find the area enclosed by the polar curve
$$r = \cos(3\theta), \quad -\frac{\pi}{6} \le \theta \le \frac{\pi}{6}$$

   (b) Set up an integral whose value is the length of the curve in part (a). Do not evaluate the integral.

5. Apply an appropriate test to determine whether the series converges conditionally, converges absolutely, or diverges:

   (a) $\sum_{n=3}^{\infty}(-1)^n \frac{\ln(n)}{n}$

   (b) $\sum_{n=1}^{\infty} \frac{n3^n}{n!}$

6. Use integration by parts to evaluate $\int x^3 e^{x^2}\,dx$.

7. Consider the parametric curve defined by
$$x = t^4 - t^2, \quad y = t^3 - t$$

(a) Find all the values of t for which the curve crosses the x-axis.

(b) Find all the points in the xy-plane where the curve crosses the x-axis.

(c) Find the slope of the curve at each time where it crosses the x-axis.

8. Consider the integral

$$\int_0^{\infty} \frac{1}{x^2 + \sqrt{x}}\, dx$$

(a) Explain why this integral is improper.

(b) Does the integral converge? (You are not required to determine the value of the integral, though you can if that is helpful.)