21-122 Integration, Differential Equations, and Approximation D. Handron

Exam #2 Reference Table

I. Trigonometric Identities

1. $\tan^2\theta + 1 = \sec^2\theta$
2. $\cot^2\theta + 1 = \csc^2\theta$
3. $\sin^2\theta = \frac{1}{2}[1 - \cos(2\theta)]$
4. $\cos^2\theta = \frac{1}{2}[1 + \cos(2\theta)]$
5. $\sin\theta\cos\theta = \frac{1}{2}\sin(2\theta)$
6. $\sin A\cos B = \frac{1}{2}[\sin(A-B) + \sin(A+B)]$
7. $\sin A\sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$
8. $\cos A\cos B = \frac{1}{2}[\cos(A-B) + \cos(A+B)]$

II. Error Estimates for Numerical Integration

The expressions below give an upper bound for approximations to $\int_a^b f(x)dx$ using the trapezoid rule, the midpoint rule, and Simpson's rule. In the expressions below K is a number such that $|f''(x)| \leq K$ for $a \leq x \leq b$ and M is a number such that $|f^{(4)}(x)| \leq M$ for $a \leq x \leq b$. The number n represents the number of subintervals into which $[a, b]$ is divided.

$$|E_T| \leq \frac{K(b-a)^3}{12n^2}$$

$$|E_M| \leq \frac{K(b-a)^3}{24n^2}$$

$$|E_S| \leq \frac{M(b-a)^5}{180n^4}$$