21-122 **Integration and Approximation** **D. Handron**

Week #2 Written Assignment: Due on Friday, May 31.

1. Find the exact length of the curve

$$y = \frac{x^4}{8} + \frac{1}{4x^2}$$

   where $1 \leq x \leq 2$

2. Solve the initial value problem

$$\frac{dy}{dx} = \frac{xy\sin(x)}{y+1}, \quad y(0) = 1$$

3. The logistic population model is described by the differential equation

$$\frac{dP}{dt} = kP\left(1 - \frac{P}{M}\right)$$

   where k and P are positive constants.

   Show that this equation has constant solutions $P(t) = 0$ and $P(t) = M$, and that other solutions are given by

$$P(t) = \frac{M}{1 + Ae^{-kt}}, \qquad \text{where } A = \frac{M - P_0}{P_0}.$$

   The constant P_0 is the initial population, $P(0) = P_0$.

4. Sketch a direction field for the differential eqation

$$y' = xy - y^2$$

   and use it to sketch a graph of the solution satisfying the initial condition $y(-1) = 1$.

5. Use Euler's method with a step size of $h = .5$ to approximate the value of the solution to the initial value problem

$$y' = xy - y^2, \quad y(-1) = 1$$

   when $x = 1$, i.e. find $(x_4, y_4) = (1, ?)$ when $(x_0, y_0) = (-1, 1)$.