Week #5 Written Assignment: Due on Friday, June 21.

1. Let $\sum a_n$ be a series with positive terms and let $r_n = a_{n+1}/a_n$. Suppose that $\lim_{n\to\infty} r_n = L < 1$, so that $\sum a_n$ converges by the ratio test. Let R_n denote the remainder after n terms, i.e.

$$R_n = a_{n+1} + a_{n+2} + a_{n+3} + \cdots = \sum_{i=n+1}^{\infty} a_i$$

   (a) Suppose that $\{r_n\}$ is a decreasing sequence. Show, by comparison with a geometric series, that

$$R_n \leq \frac{a_{n+1}}{1 - r_{n+1}}.$$

   (b) Suppose that $\{r_n\}$ is an increasing sequence. Show that

$$R_n \leq \frac{a_{n+1}}{1 - L}.$$

2. Consider the series

$$\sum_{n=1}^{\infty} \frac{2}{n3^n}$$

   (a) Using the result from the previous exercise, give an upper bout for the remainder after 10 terms, R_{10}.

   (b) How large must n be so that R_n is less than 10^{-6}?

3. Find the radius of convergence and the interval of convergence for the series

$$\sum_{n=1}^{\infty} \frac{(x+1)^n}{n4^n}.$$

4. (a) Starting with the geometric series $\sum_{n=0}^{\infty} x^n$, find the sum of the series

$$\sum_{n=1}^{\infty} nx^{n-1} \quad |x| < 1$$

   (b) Find the sum of each of the following series:

      i. $\sum_{n=1}^{\infty} nx^n \quad |x| < 1$

      ii. $\sum_{n=1}^{\infty} n\frac{1}{2^n}$

(c) Find the sum of each of the following series:

i. $\sum_{n=2}^{\infty} n(n-1)x^n \quad |x| < 1$

ii. $\sum_{n=2}^{\infty} \frac{n^2-n}{2^n}$

iii. $\sum_{n=2}^{\infty} \frac{n^2}{2^n}$

5. For the function $f(x) = \sqrt{x}$, find the Taylor series centered at $a = 4$. What is the radius of convergence of this series?

6. Find the Maclaruin series for $f(x) = x\cos(x)$. What is the radius of convergence of this series?