Week #3 Written Assignment: Due on Friday, January 31.

1. (Stewart, 7th edition, #7.4.59) The German mathematician Karl Weierstrass (1815-1897) noticed that the substitution $t = \tan(x/2)$ will convert any tational function of $\sin x$ and $\cos x$ into an ordinary rational function of t.

   (a) If $t = \tan(x/2)$, $-\pi < x < \pi$, sketch a right triangle or use trigonometric identities to show that

   $$\cos\left(\frac{x}{2}\right) = \frac{1}{\sqrt{1+t^2}} \qquad \text{and} \qquad \sin\left(\frac{x}{2}\right) = \frac{t}{\sqrt{1+t^2}}$$

   (b) Show that

   $$\cos(x) = \frac{1-t^2}{1+t^2} \qquad \text{and} \qquad \sin(x) = \frac{2t}{1+t^2}$$

   (c) Show that

   $$dx = \frac{2}{1+t^2}\,dt$$

2. (Stewart, 7th edition, #7.4.62) Using the results from problem #1, evaluate

   $$\int_{\pi/3}^{\pi/2} \frac{1}{1+\sin(x)-\cos(x)}\,dx$$

3. Evaluate the integral

   $$\int \ln(x^3+x)\,dx$$

4. (Stewart, 7th edition, #84) We know that $F(x) = \int_0^x e^{e^t}\,dt$ is a continuous function by the Fundamantal Theorem of calculus, though it is not an elementary function (see the section "Can We Integrate All Continuous Functions" in Section 7.5 of Stewart for a definition of *elementary function*).

   (a) The function $\int \frac{e^x}{x}\,dx$ is not an elementary function, but it can be expressed in terms of $F(x)$. Evaluate

   $$\int_1^2 \frac{e^x}{x}\,dx$$

   in terms of F.

(b) Similarly, the function $\int \frac{1}{\ln x}\,dx$ is not an elementary function. It too can be expressed in terms of $F(x)$. Evaluate

$$\int_2^3 \frac{1}{\ln x}\,dx$$

in terms of F.