Assignment 10 and PRACTICE FOR TEST 4, (December 6, 2004): due December 9, 2004

MATERIAL COVERED: 3.11 till (and excluding) Chapter 5

Problem 1: Prove that
(i) $f(x) = x^3 - x + 1$ has at least one real root.

(ii) there exists $c \in \mathbb{R}$ such that $f(c) = 10$, where $f(x) := x^3 - x^2 + x$.

Problem 2: Exercise 6, page 179.

Problem 3: Let

$$h(x) := \begin{cases} \tan x & \text{if } x \in \left[\frac{1}{2^{n+1}}, \frac{1}{2^n}\right], n \in \mathbb{Z} \text{ odd}, \\ x & \text{otherwise} \end{cases}$$

Prove that h is differentiable at zero.

Problem 4: Exercise 14, page 179.

Problem 5: Exercise 8, page 187.

Problem 6: Exercise 9, page 187.

Problem 7: Assume that f and f' are differentiable at every point of the interval (a, b). Let $x \in (a, b)$ and $x_0 \in (a, b)$ with $x \neq x_0$. Prove that there exists c between x_0 and x such that

$$f(x) = f(x_0) + f'(x_0)(x - x_0) + \frac{1}{2}f''(c)(x - x_0)^2.$$

Hint: Let

$$M = \frac{f(x) - f(x_0) - f'(x_0)(x - x_0)}{(x - x_0)^2}$$

and apply Rolle's theorem to

$$g(t) = f(t) + f'(t)(x - t) + M(x - t)^2.$$

Problem 8: Exercise 9, page 191.

Problem 9: Exercise 13, page 191.

Problem 10: Exercise 6, page 194.