Lecture 8

Enoch Cheung

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1. Prove $A - (B \cup C) \subseteq A - B$.

   Let $x \in A - (B \cup C)$, then $x \in A$ and $x \notin (B \cup C)$. Thus $x \notin B$, so $x \in A - B$.

2. Use double containment to prove $(A - B) \cup (C - B) = (A \cup C) - B$.

   To show $(A-B) \cup (C-B) \subseteq (A \cup C) - B$, suppose $x \in (A-B) \cup (C-B)$. Case 1: If $x \in (A-B)$, then $x \in A \subseteq A \cup C$ and $x \notin B$, so $x \in (A \cup C) - B$. Case 2: If $x \in (C - B)$, then $x \in C \subseteq A \cup C$ and $x \notin B$ so $x \in (A \cup C) - B$.

   To show $(A - B) \cup (C - B) \supseteq (A \cup C) - B$, suppose $x \in (A \cup C) - B$. Then $x \in A \cup C$ and $x \notin B$. Case 1: if $x \in A$, then $x \in A - B$. Case 2: if $x \in C$, then $x \in C - B$. In both cases, $x \in (A - B) \cup (C - B)$.

3. Let $\{B_i\}_{i\in\mathbb{N}}$ be an indexed family of sets and A be any set. Show that

$$A - \bigcap_{i\in\mathbb{N}} B_i = \bigcup_{i\in\mathbb{N}} (A - B_i)$$

   First, we show that $A - \bigcap_{i\in\mathbb{N}} B_i \subseteq \bigcup_{i\in\mathbb{N}}(A - B_i)$. Suppose $x \in A - \bigcap_{i\in\mathbb{N}} B_i$, then $x \in A$ and $x \notin \bigcap_{i\in\mathbb{N}} B_i$. If $x \in B_i$ for every $i \in \mathbb{N}$, then $x \in \bigcap_{i\in\mathbb{N}}$ which would be a contradiction, so $x \notin B_k$ for some $k \in \mathbb{N}$. This means that $x \in A - B_k$ where $k \in \mathbb{N}$, so $x \in \bigcup_{i\in\mathbb{N}}(A - B_i)$.

   Now we show that $A - \bigcap_{i\in\mathbb{N}} B_i \supseteq \bigcup_{i\in\mathbb{N}}(A - B_i)$. Suppose $x \in \bigcup_{i\in\mathbb{N}}(A - B_i)$, then for some $k \in \mathbb{N}$ we have $x \in (A - B_k)$. Thus, $x \in A$ and $x \notin B_k$. Therefore, $x \notin \bigcap_{i\in\mathbb{N}} B_i$, so $x \in A - \bigcap_{i\in\mathbb{N}} B_i$.

4. Define the symetric difference between two sets as $A \triangle B = (A - B) \cup (B - A)$. Prove the following:

$$A \cap (B \triangle C) = (A \cap B) \triangle (A \cap C)$$

   To show $A \cap (B \triangle C) \subseteq (A \cap B) \triangle (A \cap C)$, suppose $x \in A \cap (B \triangle C)$, then $x \in A$ and $x \in B \triangle C = (B - C) \cup (C - B)$. Case 1: Suppose $x \in B - C$, then $x \in B$ so $x \in A \cap B$, and $x \notin C$ so $x \notin A \cap C$. Therefore, $x \in (A \cap B) - (A \cap C)$, so $x \in (A \cap B) \triangle (A \cap C)$. Case 2: Suppose $x \in C - B$, then $x \in C$ so $x \in A \cap C$, and $x \notin B$ so $x \notin A \cap B$. Therefore, $x \in (A \cap C) - (A \cap B)$, so $x \in (A \cap B) \triangle (A \cap C)$.

   To show $A \cap (B \triangle C) \supseteq (A \cap B) \triangle (A \cap C)$, suppose $x \in (A \cap B) \triangle (A \cap C) = ((A \cap B) - (A \cap C)) \cup ((A \cap C) - (A \cap B))$. Case 1: If $x \in (A \cap B) - (A \cap C)$, then $x \in A \cap B$ and $x \notin A \cap C$, so $x \in A$ and $x \in B$, so $x \notin C$. Therefore, $x \in B - C \subseteq B \triangle C$, so $x \in A \cap (B \triangle C)$. Case 2: If $x \in (A \cap C) - (A \cap B)$, then $x \in A \cap C$ and $x \notin A \cap B$, so $x \in A$ and $x \in C$ so $x \notin B$. Therefore, $x \in C - B \subseteq B \triangle C$ so $x \in A \cap (B \triangle C)$.