Lecture 7

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1. Do there exists sets A and B such that $A - B = B - A \neq \emptyset$? Why or why not?

   No. If $A - B$ is nonempty, let $x \in A - B$, then $x \notin B$ so $x \notin B - A$. Therefore, $A - B \neq B - A$.

2. (a) Find an example of sets A, B, C such that $(A - B) - C = A - (B - C)$.

      Trivial example: $A = B = C = \emptyset$.

      Another example: $A = \{a, d\}$, $B = \{b, d, e\}$, $C = \{c, e\}$, then $(A-B)-C = A-(B-C) = \{a\}$.

   (b) Example where $(A - B) - C \neq A - (B - C)$.

      Trivial example: $A = B = C = \{x\}$.

      Interesting example: $A, B, C \subseteq \mathcal{P}(\{a, b, c\})$, $A = \{x \mid a \in x\}$, $B = \{x \mid b \in x\}$, $C = \{x \mid c \in x\}$. Then $(A - B) - C = \{\{a\}\}$ and $A - (B - C) = \{\{a\}, \{a, c\}, \{a, b, c\}\}$.

3. (a) Compare $\mathcal{P}(\{1\} \cup \{2, 3\})$ and $\mathcal{P}(\{1\}) \cup \mathcal{P}(\{2, 3\}$.

      $\mathcal{P}(\{1\} \cup \{2, 3\}) = \mathcal{P}(\{1, 2, 3\}) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{2, 3\}, \{1, 3\}, \{1, 2, 3\}\}$

      $\mathcal{P}(\{1\}) \cup \mathcal{P}(\{2, 3\}) = \{\emptyset, \{1\}\} \cup \{\emptyset, \{2\}, \{3\}, \{2, 3\} = \{\emptyset, \{1\}, \{2\}, \{3\}, \{2, 3\})$

   (b) In general, $\mathcal{P}(A) \cup \mathcal{P}(B) \subseteq \mathcal{P}(A \cup B)$.

      Suppose $X \in \mathcal{P}(A)$, then $X \subseteq A \subseteq A \cup B$, so $X \in \mathcal{P}(A \cup B)$. Same for $X \in \mathcal{P}(B)$. Thus, $\mathcal{P}(A) \cup \mathcal{P}(B) \subseteq \mathcal{P}(A \cup B)$.

4. Prove that for all sets A and B, if $A - B = \emptyset$, then $\mathcal{P}(A) \subseteq \mathcal{P}(B)$.

   If $A - B = \emptyset$, then $A \subseteq B$. Suppose $X \subseteq A$, then $X \subseteq B$. Thus $\mathcal{P}(A) \subseteq \mathcal{P}(B)$.

5. Given sets A, B, C in some universal set U, prove that if $A \subseteq B \cap \overline{C}$ then $C \subseteq \overline{A}$.

   Suppose $A \subseteq B \cap \overline{C}$, and consider $x \in C$. We claim that $x \notin A$, suppose $x \in A$, then $x \in B \cap \overline{C}$ so $x \in \overline{C}$ which contradicts $x \in C$. Therefore, $x \notin A$, so $C \subseteq \overline{A}$.