Lecture 2

Enoch Cheung

1.

Theorem 1. *If m and $m+n$ are both even integers then n is an even integer.*

The theorem is true, but the proof is not valid, because it assumes that n is even.

Proof. Suppose m and $m+n$ are even, then let $m = 2a$ and $m+n = 2b$. Then

$$\begin{aligned}(m+n) - m &= 2b - 2a \\ n &= 2(b-a)\end{aligned}$$

Thus n is even. □

2.

Proposition 1. $\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$

This proposition is true. The proof is not valid, as it assumes the proposition and derives a true statement from it. We should instead start with a true statement, and derive our proposition. We should also take care to consider values of $\theta \neq \pi\frac{k}{2} + \frac{\pi}{4}$ for $k \in \mathbb{Z}$, because for $\theta = \pi\frac{k}{2} + \frac{\pi}{4}$ the function $\tan(2\theta)$ is undefined. Luckily, the right hand side are also undefined at exactly those same points, since $\tan(\theta)$ is undefined at $\theta = \pi k + \frac{\pi}{2}$ and $\tan(\theta) = 1$ when $\theta = \pi k + \frac{\pi}{4}$

To fix this proof, we notice that each implication in the original proof are in fact "if and only if" (double implication). The only potential problem is when dividing $\cos^2(\theta)$, but we see that it is nonzero for all the points we are considering, since $\cos(\theta) = 0 \iff \theta = \pi k + \frac{\pi}{2}$.

3.

Theorem 2. *If $x > 0$ and $y > 0$ then $x + y > 0$.*

We assume that x and y are both positive, and conclude that $x+y$ is positive.

Proof. Suppose $x > 0$ and $y > 0$, then

$$\begin{aligned}x &> 0 \\ x + y &> y \\ x + y &> y > 0 \\ x + y &> 0\end{aligned}$$

□

4. The proof removed squares without consider both square roots.